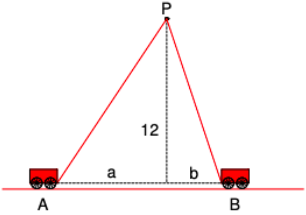
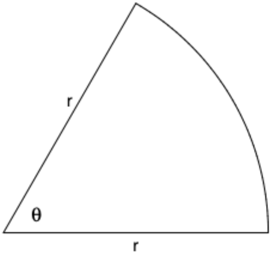
2010 final exam

(10 points) 4. Two carts A and B are connected by a rope 39 metres long that passes over a small stationary pulley P, 12 metres above the height at which the rope is attached to the carts, as shown in the figure. The rope is assumed to form straight lines from the carts to the pulley, with constant total length 39 metres. Cart B is pulled to the right at 2 metres per second. How fast is cart A moving when the point where the rope is attached to cart B is 5 metres to the right of the pulley? *Hint:* If the sum of two things is constant, how are their rates of change related?



(10 points) 7. A sector of a circle with radius r and opening angle θ has area $A = r^2\theta/2$. Find r and θ for the sector with smallest perimeter, given that the area $A = 9$. Note that the perimeter consists of two radii and a circular arc. Be sure to verify that your answer is the global minimum.



(5 points) 8. After a certain drug is injected into a person's bloodstream, its concentration in the blood decreases at a rate proportional to the existing concentration. If time is measured in hours, the constant k in the differential equation for the concentration is 0.25. If the initial concentration was 0.8 mg of drug per ml of blood, how long does it take for the concentration to decrease to 0.16 mg per ml?

2011 final exam

Math 102 **Name:** ____________________

5. (10 points) Find the absolute maximum and absolute minimum values of $f(x) = \sin(x) - \cos(x)$ on $[0, 2\pi]$.

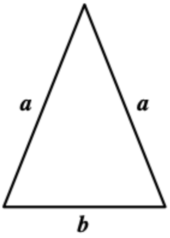
Math 102 **Name:** ____________________

6. (9 points) Let $f(x) = \ln(x^3)$ and $g(x) = (\ln x)^3$. Find *all* values of x where the tangent lines of these two functions have the same slope (or show that no such values exist).

Math 102 **Name:** ____________________

9. (10 points) Consider the isosceles triangle given in the following figure, where a and b indicate the side lengths. Assume that the triangle has a circumference of 2. Find the lengths a and b for which the area A of the triangle is maximized. You must also check that you found a maximum and your solution must include that check.

   Hint: Heron's formula may be useful, which states that $A^2 = s(s-x)(s-y)(s-z)$, where A is the area of the triangle, x, y, z its side lengths and $s = \frac{x+y+z}{2}$.



2012 final exam

C.2 [4 pts] Use the definition of the derivative to calculate the derivative of $g(x) = \sqrt{x}$. The following might be a useful fact in simplifying the limit: $(\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b}) = a - b$.

C.5.1 [8 pts] A hot air balloon with a basket hanging below it is released from the ground and rises straight up at a speed of 5 meters per second. At the moment the balloon is released, a girl is 29 meters from the point on the ground directly below the balloon and is riding her bicycle toward the ballon at a speed of 2 meters per second. From the perspective of the girl, at what time t is the angle between the ground and the basket hanging below the balloon changing most quickly?

C.5.2 [8 pts] An architect is designing a house in the form of a cylinder covered by a roof in the shape of half a sphere (extending above the cylinder). Suppose the material used to build the cylindrical wall is half the price of the material that is used to build the roof per unit area. If the total volume of the house is fixed, what ratio between the height of the wall and the radius of the roof will minimize the cost?