Radioactive decay

Consider a small time period from t to $t+h$.
Number of atoms that have decayed $= khQ(t)$.

$$Q(t+h) = Q(t) - khQ(t)$$

mass balance.

$$\Rightarrow \quad \frac{Q(t+h)-Q(t)}{h} = -kQ(t)$$

$$\lim_{h\to 0} \frac{Q(t+h)-Q(t)}{h} = Q'(t) \quad \text{(by definition)}$$

$$\Rightarrow \quad Q'(t) = -kQ(t)$$

All solutions to this differential equation have this form (general solutions)

$$\Rightarrow \quad Q(t) = Ce^{-kt} \quad \text{for some } C$$

If we know $Q(0) = Q_0$ then

$$Q(0) = Ce^{-k\cdot 0} = C = Q_0 \quad \Rightarrow \quad Q(t) = Q_0 e^{-kt}$$

specific solution.

$k>0$. $\rightarrow$ $Q(t)$ exponentially decays.

Half-life.

At what time $t_{1/2}$ is $Q(t_{1/2}) = \frac{1}{2}Q(0)$?

Notice $Q(t) = Q_0 e^{-kt}$

plug in $Q(t) = \frac{1}{2}Q_0$, solve for t.

$$\frac{1}{2}Q(0) = \frac{1}{2}\cancel{Q_0} = \cancel{Q_0}\, e^{-k \cdot t_{1/2}}$$

$$\Rightarrow e^{-kt_{1/2}} = \frac{1}{2}$$

$$2 = e^{kt_{1/2}}$$

take ln on both sides (and ~~switch~~ left/right)

switch

$$kt_{1/2} = \ln 2$$

$$t_{1/2} = \frac{\ln 2}{k}$$

Remark: ① $t_{1/2}$ is independent of Q_0.

↳ half-life.

② $k\uparrow$ → atoms are more radioactive

→ decay faster → shorter half-time.

model prediction matches our expectation!