Inverse Functions and Logarithms

Math 102 Section 102
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Due due due due due...

- Oct 29 (Today): Pre-lecture 9.1
- Oct 31 (Wednesday): Pre-lecture 9.2
- Nov 1 (Thursday): Assignment 8

Assignments due: 9:00 pm

Today: learning goals

1. Explain the definition of an inverse function and $\ln(x)$
2. Calculate the derivative of $\ln(x)$ and a^x
3. Apply logarithms to solve application problems (moved to Wednesday)

Inverse function

Definition (Inverse function)

Given a function $y = f(x)$, its inverse function, denoted as f^{-1}, satisfies

$$f^{-1}(f(x)) = x.$$

Q1. $f(f^{-1}(x)) =$

A. x

B. $-x$

C. $\frac{1}{x}$

D. $-\frac{1}{x}$

E. depends on $f(x)$

Issue of existence

Q2. The inverse function of $f(x)$ is the mirror image of the graph of $f(x)$ in the line $y = x$.

A. True.

B. False.

- Not every function has an inverse function.
- If a function $f(x)$ has an inverse, then for each value y in its range, there is one and only one x such that $f(x) = y$.
- Such a function is called a one-to-one (or 1-1 or bijective) function.

Geometry of inverse functions

- The graphs of inverse functions are symmetric about the line $y = x$.
- $\forall(x, y)$ on the graph of $f(x)$, (y, x) is on the graph of $f^{-1}(x)$. Same vice versa.

https://www.desmos.com/calculator/gweahslmzx

Logarithm

Definition (Logrithm)

$y = \log_a x$ is the inverse function of the exponential function $y = a^x$ $(a > 0)$. In particular, denote $\log_e x$ as $\ln x$ (sometimes also written as $\log x$).

Bacteria colony

Q3. Starting from a single cell, how long will it take for an *E. coli.* colony to reach size of $6 \cdot 10^8$ cells by doubling every 20 minutes?

Note that the if time is measured in minutes, the number of bacteria at time t is

$$B(t) = 2^{\frac{t}{20}}.$$

A. $t = 2\frac{\ln(8)+6\ln(10)}{\ln(20)}$

B. $t = 20\frac{\ln(8)+6\ln(10)}{\ln(2)}$

C. $t = 2\frac{\ln(6)+8\ln(10)}{\ln(20)}$

D. $t = 20\frac{\ln(6)+8\ln(10)}{\ln(2)}$

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- Derivative of a^x revisited
- Derivative of $\log_a x$

Answers

1. A
2. B
3. D

Related Exam Problems

1. Consider the function $f(x) = \log_2(x)$. Find the slope of this function at the point $x = 1$.
2. If $f(x) = x^x$, find $f'(x)$.
3. Let f^{-1} be the inverse function of $f(x)$. Assume $f(0) = 1$ and $f'(0) = 2$. Find the tangent line $y = mx + b$ to $f^{-1}(x)$ at 1.
4. If $f(x) = x^{x^x}$, find $f'(x)$ (not really an exam problem, but interesting)