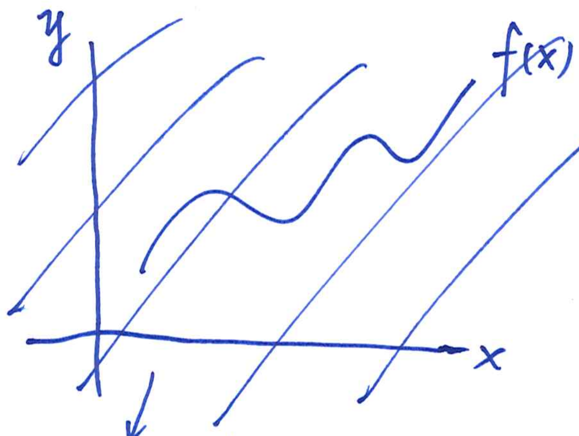
Oct. 29
Monday

1. $\frac{d}{dx} e^{kx}$

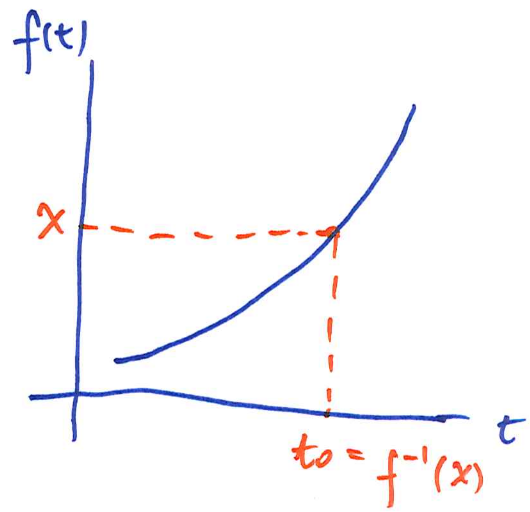
$$\frac{d}{dx} e^{kx} = e^{kx} \cdot \frac{d}{dx}(kx) = k e^{kx}$$

chain rule

2. (Q1). $f(f^{-1}(x))$

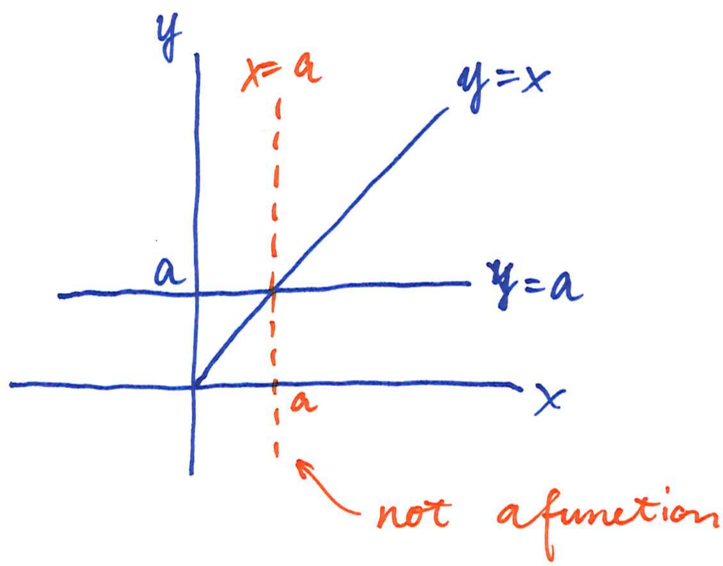


Not a good example.
Inverse does not exist!
(see below)



$f(f^{-1}(x)) = f(t_0) = x$

3. (Q2). (geometry, issue of existence)



not a function

4. (Q3) E. coli growth.

$$B = 2^{\frac{t}{20}}$$

$$\ln B = \ln(2^{\frac{t}{20}}) = \frac{t}{20}\ln 2$$

We know $B = 6 \cdot 10^8$. $\ln B = \ln(6 \cdot 10^8)$
$= \ln 6 + 8 \ln 10$

$$\Rightarrow \quad \ln 6 + 8\ln 10 = \frac{t}{20}\ln 2$$

$$\Rightarrow \quad t = 20\,\frac{\ln 6 + 8\ln 10}{\ln 2}$$

5. Derivative of a^x $(a>0)$

Recall $\frac{d}{dx} a^x = C_a a^x$

↳ What's this coefficient?

→ Logs can help us!

Set $y = a^x$. $\ln y = x \ln a$

→ implicit differentiation:

$$\frac{1}{y}\frac{dy}{dx} = \ln a \Rightarrow \frac{dy}{dx} = y \ln a = \underbrace{(\ln a)}_{C_a} a^x$$

↳ requires derivative of natural logs

An alternative that does not require differentiating natural logs:

Notice $a^x = (e^{\ln a})^x = e^{(\ln a)x}$

$$\frac{d}{dx}\left(e^{(\ln a)x}\right) = (\ln a)\, e^{(\ln a)x}$$ ← see P1. today.

$$= \underbrace{(\ln a)}_{C_a} a^x$$

yes! agrees!

6. Derivative of $y = \log_a x$

Let $f(x) = \log_a x$. Then $a^{f(x)} = x$

Implicit differentiation

$$\underbrace{(\ln a)\, a^{f(x)}}_{\text{see P5 above.}} \cdot \underbrace{\frac{d}{dx} f(x)}_{\text{chain rule}} = 1$$

$$\frac{df}{dx} = \frac{1}{\ln a}\,\frac{1}{a^{f(x)}}$$

$$= \frac{1}{\ln a}\,\frac{1}{x}$$

In particular, if $a=e$, then $\frac{d}{dx}(\ln x) = \frac{1}{x}$