**Exponential Functions** 

Math 102 Section 102 Mingfeng Qiu

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# Midterm lesson

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*	Also, does anyone know where I can find answer keys to the sample exams? Not sure why we don't have access to em in the first place but if not I guess I have to improvise.	SUBSCRIBE
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Assignment 7 is due tonight at 9:00.

## Where are we going next?

- Recall: predator and prey.
  - Prey population: x
  - Prey reproduction rate: rx
  - Predation rate:  $\frac{Kx}{a+x}$
- A new type of equation.

$$\frac{dx}{dt} = rx - \frac{Kx}{a+x}$$

- A differential equation relates one or more derivatives of a function to the function itself.
- Goal for the following three weeks:
  - solve them;
  - understand the structure of their solutions without solving them.

We are going to do some preparation before going into differential equations.

- 1. Identify exponential functions
- 2. Calculate derivatives of exponential functions
- 3. Define the number e

- Q1. Which of the following is an exponential function?
  - A.  $x^n$  power function
  - B.  $2^x$  variable in the exponent!
  - C.  $e^2$  this is a constant
  - D.  $\ln(x)$  Inverse of exp. function

Q2. The number of the bacteria *E. coli* in a culture doubles every twenty minutes. Starting with one *E. coli* cell, find the size of the population after 3 hours and 20 minutes.

- A. 32
- **B**. 64
- C. 1024
- D. 2048

3 hours and 20 minutes  $\Rightarrow$  10 generations  $\Rightarrow$  size is  $2^{10} = 1024$ 

# Exponential growth

Q3. Following the previous question, how big is the population after one day?

- **A**. 2048
- **B**.  $2^{24}$
- C.  $2^{36}$
- D.  $2^{72}$

1 day  $\Rightarrow 3 \times 24 = 72$  generations  $\Rightarrow$  size is  $2^{72}$ 

Mass of one *E. coli* cell:  $10^{-12}$  kg.  $2^{72}$  cells weigh 4.7 million tons.

In two days of time, the total mass of the cells will be more than 1 million planets of Earth.

- Q4. Exponential functions  $a^x$ , where a > 1...
  - A. All go through the point (1,1)
  - B. All go through the point (1,0)
  - C. If a < b, then  $a^x < b^x$  for all x > 0 and  $a^x > b^x$  for all x < 0
  - D. If a < b, then  $a^x < b^x$  for all x > 1 and  $a^x > b^x$  for all x < 1
  - E. None of the above

## Exponential functions

If a < b, then  $a^x < b^x$  for all x > 0 and  $a^x > b^x$  for all x < 0.





### Exponential functions

Q5. The derivative of  $f(x) = a^x$  is A.  $f'(x) = xa^{x-1}$ B.  $f'(x) = ax^{a-1}$ C.  $f'(x) = \frac{a^{x+h}-a^x}{h}$ D.  $f'(x) = a^x$ E.  $f'(x) = Ca^x$ 

### Document camera

- Derivative of  $f(x) = a^x$
- Why  $e^x$ ?

#### Definition (e)

The number e is defined as

$$e = \lim_{h \to 0} \left(1 + h\right)^{\frac{1}{h}}$$

or equivalently

$$e = \lim_{n \to \infty} \left( 1 + \frac{1}{n} \right)^r$$

Remark: e is defined such that the derivative of the exponential function  $f(x) = e^x$  is itself.

# Example (document camera)

#### Calculate



- Exponential functions different from power functions
- Exponential growth (e.g. bacterial cultures)

• If 
$$f(x) = a^x$$
, then  $f'(x) = C_a a^x$ 

▶  $e^x$  is a special exponential function such that  $\frac{d}{dx}e^x = e^x$ 

## Answers

B
 C
 D
 D
 C
 E

1. Sketch the graph of the function  $y = x^2 e^{-x}$