Oct 26. Doc cam.

Derivative of $f(x) = a^x$ $(a>0)$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{a^{x+h} - a^x}{h}$$

$$= \lim_{h\to 0} \frac{a^x(a^h - 1)}{h}$$

$$= a^x \underbrace{\left(\lim_{h\to 0} \frac{a^h - 1}{h}\right)}$$

constant once evaluated.
Call it C_a
(Actually $C_a = \ln a$)

→ Don't like C_a. Can we make it 1, by a smart choice of a?

If $C_a = 1$, then $C_a = \lim_{h\to 0} \frac{a^h - 1}{h} = 1$

⇒ When h is small, $\frac{a^h - 1}{h} \approx 1$

$$a^h \approx 1 + h$$

$$a \approx (1+h)^{1/h}$$

What about $\lim_{h \to 0} (1+h)^{1/h}$

h	$a \approx (1+h)^{1/h}$
0.1	2.5937
0.01	2.704813
0.001	2.716923
$\vdots$	$\vdots$
0.00001	2.7182682372

$\hookrightarrow (1+h)^{1/h}$ approaches a number

It turns out that this number is the right value for a!

Call it e.

Remark. e is defined such that $f(x) = e^x$ has the derivative as itself, i.e. $f'(x) = f(x) = e^x$.