Implicit Differentiation

Math 102 Section 102
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Annoucements

- Please fill out the OSH survey
- Link: https://ubc.ca1.qualtrics.com/jfe/form/SV_8plWbfttDzatF1X

Exam tips

- It's perfectly OK to feel nervous. Just keep calm and work with it.
- Don't linger on a difficult problem for too long.
- Be careful of details.
- When checking your answers, use intuition, alternative solution etc.

Today: learning goals

- Identify implicit functions defined through relations.
- Describe implicit differentiation geometrically.
- Perform implicit differentiation using chain rule.

Implicit differentiation

- Sometimes you don't want to or can't isolate a function whose derivative is required.
- e.g. Find the equation of the tangent line to $x^2 + y^2 = 25$ at (x_0, y_0).
- e.g. What is the highest point on the ellipse $x^2 + 3y^2 - xy = 1$?
- Let $y = y(x)$ and differentiate implicitly:

$$x^2 + y(x)^2 = 25 \Rightarrow 2x + 2y\frac{dy}{dx} = 0$$

Warm-up for implicit Differentiation

Q1. If $x^2 + y^2 = r^2$, find $\frac{dy}{dx}$.

A. $\frac{dy}{dx} = \frac{-x}{y}$

B. $\frac{dy}{dx} = \frac{-y}{x}$

C. $\frac{dy}{dx} = \frac{x}{y}$

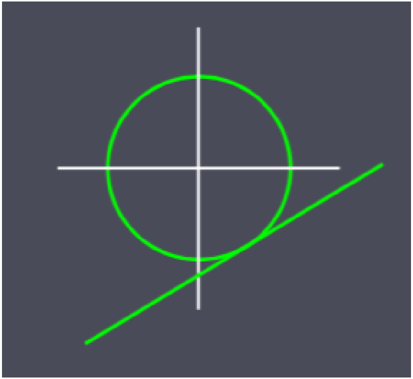
D. $\frac{dy}{dx} = \frac{y}{x}$

$$2x + 2y\frac{dy}{dx} = 0 \Rightarrow x = -y\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{-x}{y}$$

Remark: This is only valid while $y \neq 0$!

The geometry of implicit differentiation

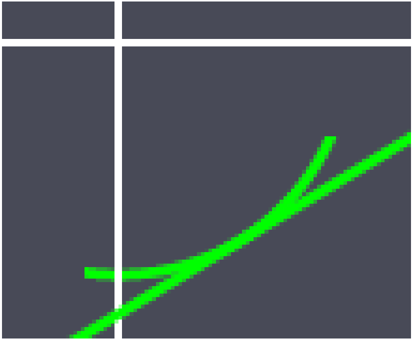
Find the tangent line to the curve defined by $x^2 + y^2 = 25$ at $(3, -4)$.



- This is not even a function!
- Which derivative are we taking?

The geometry of implicit differentiation

Find the tangent line to the curve defined by $x^2 + y^2 = 25$ at $(3, -4)$.



- Near the point $(3, -4)$, y is a function of x:

$$x^2 + y(x)^2 = 25$$

- Thus,

$$2x + 2y\frac{dy}{dx} = 0$$

The geometry of implicit differentiation

Find the tangent line to the curve defined by $x^2 + y^2 = 25$ at $(3, -4)$.

- Near the point $(3, -4)$, y is a function of x:

$$x^2 + y(x)^2 = 25$$

- Thus,

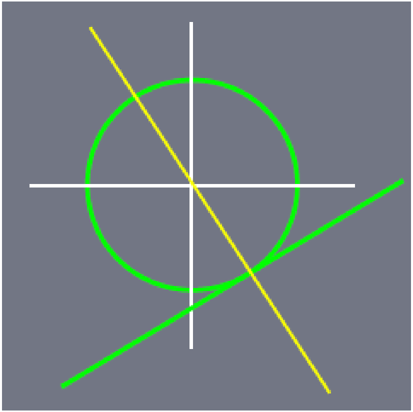
$$2x + 2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{-x}{y}$$

- Equation of tangent line:

$$y + 4 = \frac{3}{4}(x - 3)$$

Tangent lines are perpendicular to the radius

$$\frac{dy}{dx} = \frac{-x}{y} = -\left(\frac{1}{y/x}\right)$$



Examples

1. Find the derivative of y with respect to x if $e^y + 2xy = \sqrt{3}$.
2. Use implicit differentiation to calculate the derivative of the function $f(x) = x^{\frac{n}{m}}$ where n and m are integers.

Implicit Differentiation

Q2. Use implicit differentiation to find $\frac{dy}{dx}$ if

$$-x^2 + 4xy = \pi$$

A. $\frac{dy}{dx} = \frac{1}{x}\left(\frac{x}{2} - y\right)$

B. $\frac{dy}{dx} = \frac{1}{x}\left(\frac{x}{4} - y\right)$

C. $\frac{dy}{dx} = \frac{1}{y}\left(\frac{x}{2} - y\right)$

D. $\frac{dy}{dx} = \frac{1}{y}\left(\frac{x}{4} - y\right)$

$$-2x + 4\left(x\frac{dy}{dx} + y\right) = 0 \Rightarrow x\frac{dy}{dx} + y = \frac{x}{2}$$

Summary

- To find the derivative of relations (not functions!) use implicit differentiation.
- Zooming in on a point of the curve shows that a function can be constructed $y = y(x)$.
- Tangent lines to a circle are always perpendicular to the radius.
- The power rule holds for fractional powers. If $y = f(x) = x^{m/n}$, then

$$\frac{dy}{dx} = \frac{m}{n} x^{\frac{m}{n}-1}.$$

May it be...

Good luck with your midterm!!!

Answers

1. A
2. A

Related Exam Problem

1. Consider the curve whose equation is

$$x^6 - 3xy + y^6 = 1.$$

   Find the slope of the tangent line at the point $(1, 0)$ on this curve. Determine whether the curve is concave up or concave down at the point $(1, 0)$.

Extra practice problems

- Online textbook Example 9.9