Oct 24 ①

with respect to

1. Find the derivative of y w.r.t. x
if $e^y + 2xy = \sqrt{3}$

$\frac{d}{dx}$: $\frac{dy}{dx}e^y + \left(2y + 2x\frac{dy}{dx}\right) = 0$

Note: chain rule : $\frac{d}{dx}(e^y) = e^y \cdot \frac{dy}{dx} = \frac{dy}{dx}e^y$

product rule : $\frac{d}{dx}(2xy) = 2 \cdot y + 2x \cdot \frac{dy}{dx}$

$= 2y + 2x\frac{dy}{dx}$

$\Rightarrow (e^y + 2x)\frac{dy}{dx} = -2y$ ← isolate $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{-2y}{e^y + 2x}$$

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2. Use implicit differentiation to calculate

$$\frac{d}{dx} f(x) = \frac{d}{dx}(x^{n/m}),$$

where n, m are integers.

Let $y = f(x) = x^{\frac{n}{m}}$

$$y^m = (x^{\frac{n}{m}})^m = x^n$$

$$\frac{d}{dx}: \quad my^{m-1} \cdot \frac{dy}{dx} = nx^{n-1}$$

$$\frac{dy}{dx} = \frac{nx^{n-1}}{m} y^{1-m}$$

We know $y = x^{\frac{n}{m}}$, so

$$\frac{dy}{dx} = \frac{nx^{n-1}}{m} x^{\frac{n(1-m)}{m}}$$

$$= \frac{n}{m} x^{n-1+\frac{n(1-m)}{m}}$$

$$= \frac{n}{m} x^{n-1+\frac{n}{m}-n}$$

$$= \frac{n}{m} x^{\frac{n}{m}-1}$$

Yes!. Power rule works for fractional powers!