Related Rates

Math 102 Section 102
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Oct. 22, 2018

Due due due due due...

- Oct 22 (Today): Pre-lecture 8.1
- Oct 24 (Wednesday): Pre-lecture 8.2
- Oct 26 (Friday): Assignment 7

Assignments due: 9:00 pm

Today

- Related rates

Related Rates

- When two quantities, Q_1 and Q_2, are related to each other, if one changes in time so will the other.
- The relationship between Q_1 and Q_2 will give you the relationship between $\frac{dQ_1}{dt}$ and $\frac{dQ_2}{dt}$:

$$\text{If } Q_2 = f(Q_1), \text{ then } \frac{dQ_2}{dt} = \frac{df}{dQ_1}\frac{dQ_1}{dt}.$$

Example

If $z^2 = x^2 + y^2$, $z > 0$, $\frac{dx}{dt} = 4$, $\frac{dy}{dt} = 5$, find $\frac{dz}{dt}$ when $x = 5$ and $y = 12$.

Q1. True or false?

$$\frac{dz}{dt} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{z}$$

A. True

B. False

Example

If $z^2 = x^2 + y^2$, $z > 0$, $\frac{dx}{dt} = 4$, $\frac{dy}{dt} = 5$, find $\frac{dz}{dt}$ when $x = 5$ and $y = 12$.

$$\frac{dz}{dt} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{z}$$

Q2. What is z when $x = 5$ and $y = 12$?

A. 0

B. 13

C. 17

D. 169

$$z^2 = 5^2 + 12^2 = 25 + 144 = 169$$

Example

If $z^2 = x^2 + y^2$, $z > 0$, $\frac{dx}{dt} = 4$, $\frac{dy}{dt} = 5$, find $\frac{dz}{dt}$ when $x = 5$ and $y = 12$.

$$\frac{dz}{dt} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{z}$$

$z = 13$ when $x = 5$ and $y = 12$.

Q3. What is $\frac{dz}{dt}$, when $x = 5$ and $y = 12$?

A. 13/80

B. 80

C. 80/13

D. 13

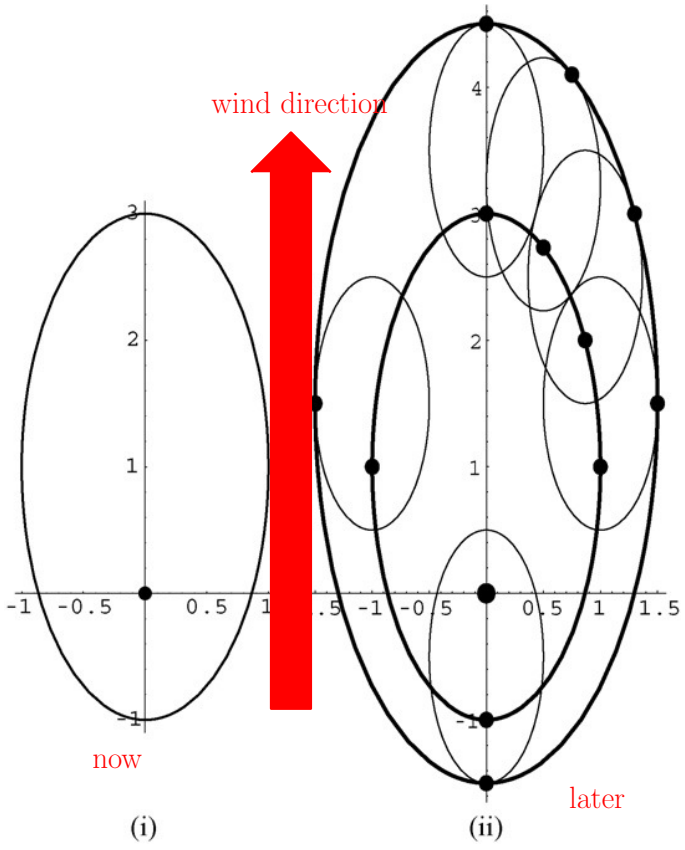
$$\frac{dz}{dt} = \frac{5 \cdot 4 + 12 \cdot 5}{13} = \frac{80}{13}$$

Related rates

Document Camera Examples

1. The radius of a spherical tumour grows at a constant rate k. Determine the rate of growth of the volume of the tumour when the radius is 1 cm.
2. Water is leaking out of a conical cup of height H and radius R. Find the rate of change of the height of water in the cup when the cup is full, if the volume of water is decreasing at a constant rate k.

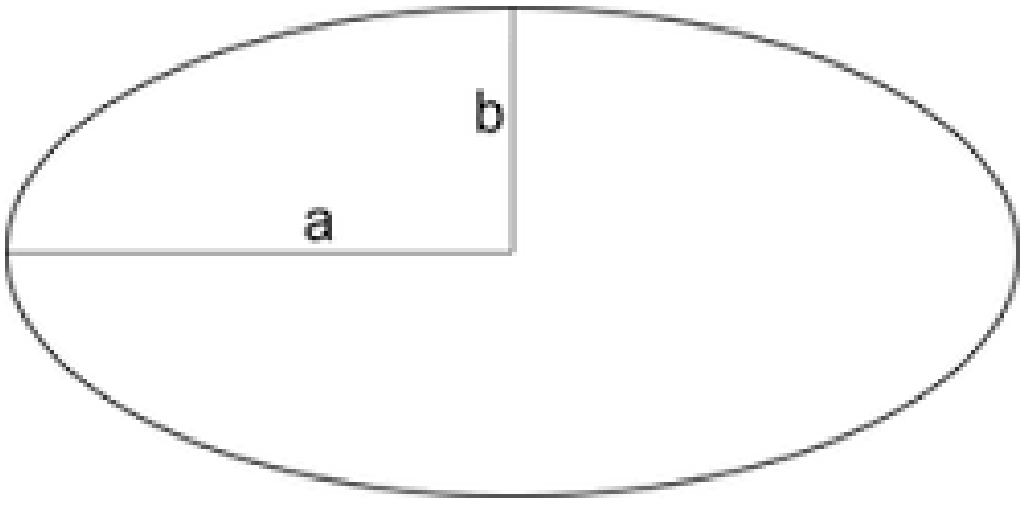
Related rates: forest fire



Jan Glasa, Ladislav Halada, On elliptical model for forest fire spread modeling and simulation, Mathematics and Computers in Simulation, Volume 78, Issue 1, June 2008, Pages 76-88

Related Rates: forest fire

A forest fire is the shape of an ellipse with semi-major axis a and semi-minor axis b:



The area of this fire is $A = \pi ab$.

Related Rates: forest fire

$$A = \pi ab.$$

Due to wind, $\frac{da}{dt} = 1$ m/min, and $\frac{db}{dt} = \frac{1}{4}$ m/min. The rate of change of the area of the fire when $a = 100$ m and $b = 80$ m is

Q4.

A. π m^2/min

B. 100π m^2/min

C. 105π m^2/min

D. 120π m^2/min

$$\frac{dA}{dt} = \pi\left(b\frac{da}{dt} + a\frac{db}{dt}\right) \Rightarrow \left.\frac{dA}{dt}\right|_{(100,80)} = \pi(80 \cdot 1 + \frac{100}{4})$$

Summary

- Related Rates: The relationship between Q_1 and Q_2 also gives the relationship between Q_1' and Q_2'.
- To solve a Related Rates problem:
  1. Sketch & formulate an idea
  2. Find an appropriate equation describing the relations between quantities
  3. Relate the rates (differentiate...)
  4. Substitute appropriate values
  5. Reality check

Answers

1. A
2. B
3. C
4. C

Related Exam Problem

1. Oil is leaking out of a cargo ship at a rate of 1 m^3/hr, forming a circular patch on the surface of the water. The radius $r(t)$ of the oil slick increases while its thickness, $\tau = 0.01$ m, remains constant. Find the rate of change of the radius at the moment when $r = 10$ m.