The radius of a spherical tumor grows at a constant rate K. Determine the rate of growth of the volume of the tumor when the radius is 1 ~~cm~~.

radius ↑

volume ↑

setup.
expectation

$\frac{dr}{dt} = K$. want $\frac{dV}{dt}$

$V = \frac{4}{3}\pi r^3$

Identify relationship

$$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \cdot \frac{dr}{dt}$$

Derivative.
- chain rule

$$= \frac{4}{3}\pi \cdot 3 \cdot 1^2 \cdot K$$

plug. in

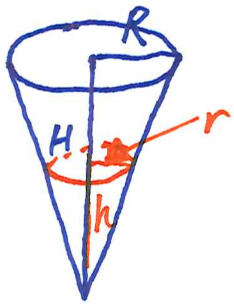
$$= 4\pi K$$

→ Yes!
agrees with expectation.

reality check.

Water is leaking out of a conical cup of height H and radius R. Find the rate of change of the height of water in the cup when the cup is full, if the volume of water is decreasing at a constant rate K.

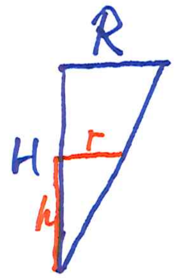
(formula: volume of a cone: $\frac{1}{3}\pi R^2 H$)



When h is large, h decreases slower. When h is small, h decreases faster.

The volume of the water: $V = \frac{1}{3}\pi r^2 h$

Both r, h are variables in time. want to get rid of one. How?



similar triangles.

$$\frac{R}{H} = \frac{r}{h} \Rightarrow r = \frac{R}{H} h$$

$$\Rightarrow V = \frac{1}{3}\pi\left(\frac{R}{H}h\right)^2 h$$

$$= \frac{1}{3}\pi\frac{R^2}{H^2}h^3$$ ← now a function of only one variable.

$$\frac{dV}{dt} = \frac{1}{3}\pi\frac{R^2}{H^2}\cdot 3h^2\cdot\frac{dh}{dt}$$

$$-K = \frac{\pi R^2}{H^2}\cdot h^2\frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{-KH^2}{\pi R^2 h^2}$$

When the cup is full, $h = H$

$$\frac{dh}{dt} = \frac{-KH^2}{\pi R^2\cdot H^2} = -\frac{K}{\pi R^2}$$

matches intuition.