# Optimal foraging: a model for ecology

Math 102 Section 102
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Oct. 15, 2018

# Due due due due due...

- Oct 15 (Today): Pre-lecture 7.1
- Oct 17 (Wednesday): Pre-lecture 7.2
- Oct 18 (Thursday): Assignment 6
- Oct 19 (Friday): OSH 4

Assignments due: 9:00 pm

# Last week

Cell division, logistic growth, baculovirus, Kepler's wedding

To solve an application problem in optimization:

- Identify the objective function and constraints.
- Use the constraints to eliminate extra variables in order to write the objective function in term of only one independent variable.
- Use calculus to find extrema.
- Describe the conclusion in the context of the application.

Today: optimal foraging - building a model for ecology

# Recall: from OSH 2

Bears search for berries that grow in patches that can be spread out across a large area. A bear will spend time in one patch gathering food before moving to another patch. The number of berries collected in a patch depends on the amount of time spent in the patch:

$$B(t) = \frac{At}{k+t}$$

https://www.desmos.com/calculator/cwkxvbdpj2

## Behavioural Ecology

Hypothesis:

- Organisms need to surive and reproduce despite ecological pressures
- Evolution selects for those animals with optimal behaviour

What should an organism optimize?

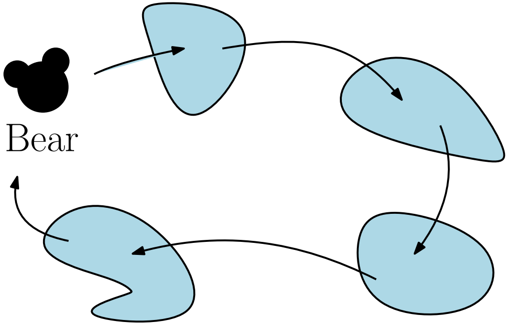
# Optimal Foraging

- Optimization of food intake.
  - Collect the most food per unit time = average rate of energy gain

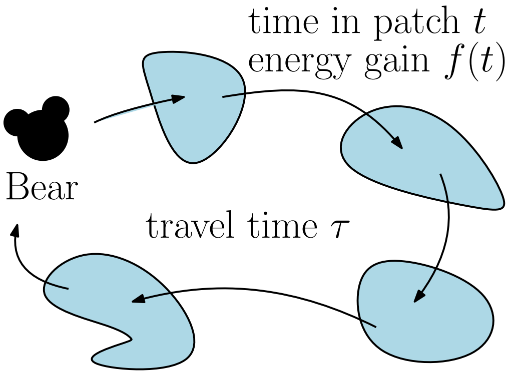
$$R = \frac{\text{energy gained}}{\text{total time spent}}$$

# How long should I stay in a food patch?

- Time is limited!
- How long should the bear stay in the patch eating berries, when it takes some time to move between patches of berries?



# How long should I stay in a food patch?

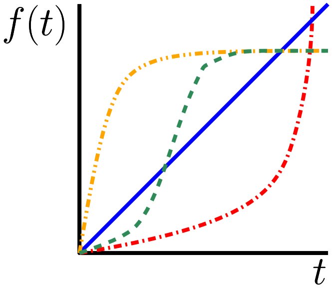



- $\tau$ = travel time
- $t$ = time spent in the food patch
- $f(t)$ = energy obtained during time $t$

# Energy gain $f(t)$

Q1. Which of the following matches the given description of energy gain?

Collection is proportional to the amount of time I spend in the patch.

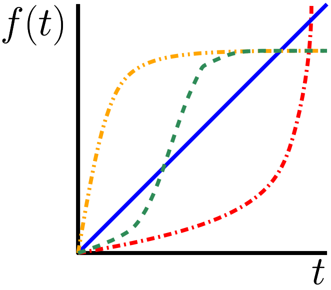



A. Blue (solid)
B. Red (dash dot)
C. Green (dash)
D. Orange (dash dot dot)

# Energy gain $f(t)$

Q2. Which of the following matches the given description of energy gain?

Collection goes well at first but gradually goes down as the resource is depleted.




A. Blue (solid)

B. Red (dash dot)

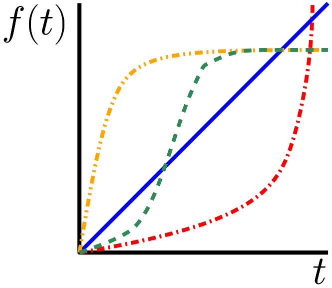
C. Green (dash)

D. Orange (dash dot dot)

# Energy gain $f(t)$

Q3. Which of the following matches the given description of energy gain?

Collection is initially difficult but becomes easier. Eventually there is no more food left.




A. Blue (solid)
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C. Green (dash)
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# Average energy gain

Consider the process of a bear traveling to one patch of berries, and spending time $t$ on foraging in the patch.

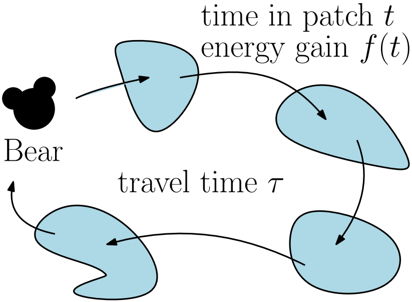
- Average energy gain per unit time:

$$R(t) = \frac{\text{energy gained}}{\text{total time spent}}$$

- energy gained $= f(t)$
- total time spent $=$ travel time $+$ time at patch $= \tau + t$

$$R(t) = \frac{f(t)}{t+\tau}.$$

# Bear eating berries




$$f(t) = B(t) = \frac{At}{k+t}$$

$$\Rightarrow R(t) = \frac{At}{(\tau+t)(k+t)}$$

# Bear eating berries

Mathematical model:

- Maximize

$$R(t) = \frac{At}{(\tau + t)(k + t)}$$

# Bear eating berries

Sketch

$$R(t) = \frac{At}{(\tau + t)(k + t)} = \frac{At}{\tau k + (\tau + k)t + t^2}$$

- When $t = 0$, $R = 0$.
- For small $t$: $R \approx \frac{A}{\tau k} t$ (straight line)
- For large $t$: $R \approx \frac{A}{t}$

# Bear eating berries

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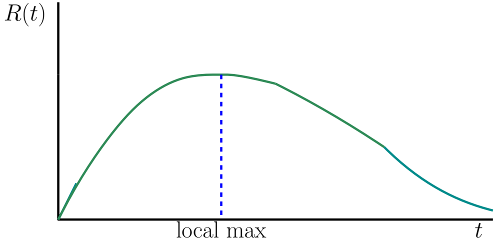


# Bear eating berries

- Maximize $R(t) = \frac{At}{(\tau+t)(k+t)}$
- Set the derivative to zero:

$$R'(t) = A\frac{k\tau - t^2}{(k+t)^2(\tau+t)^2} = 0$$

- Critical points: $k\tau - t^2 = 0 \Rightarrow t_{1,2} = \pm\sqrt{k\tau}$
- Keep the positive root: $t = \sqrt{k\tau}$.

# Bear eating berries

Q4. Are we done? $t = \sqrt{k\tau}$.

A. Yes, we have found the optimal time.

B. No, we still have to compute $R(t)$ for this time and check that it is larger than $R(0)$.

C. No, we need to check if there is a constraint to satisfy.

D. No, we have to check that we found a local maximum.

# Bear eating berries

Always check the type of critical point!

- Sketch the function (to get an idea, may not be accurate)
- Use FDT:
  - When $t < \sqrt{k\tau}$, $R'(t) > 0$.
  - When $t > \sqrt{k\tau}$, $R'(t) < 0$
- or use SDT, and check that $R''(t) < 0$ for $t = \sqrt{k\tau}$.

# Bear eating berries

Q5. $t = \sqrt{k\tau}$.

If it takes the bear a long time to get to the patch, to maximize the average energy gain, the bear should

A. Stay in the patch for a longer time

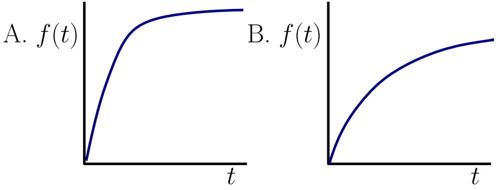
B. Stay in the patch less time

If $\tau$ is large, then the optimal time to stay in the patch $t = \sqrt{k\tau}$ is also large.

# Bear eating berries

Q6. $t = \sqrt{k\tau}$.

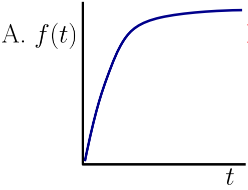
There are two different patches which take the same amount of time $\tau$ to get to. In which patch should the bear spend more time in order to maximize the average energy gain?



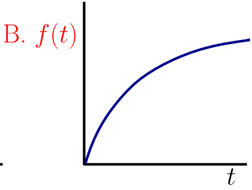

$f(t) = \frac{t}{20+t}$ $\qquad$ $f(t) = \frac{t}{200+t}$

# Bear eating berries

Q6. Recall that $t = \sqrt{k\tau}$.
There are two different patches which take the same amount of time $\tau$ to get to. In which patch should the bear spend more time in order to maximize the average energy gain?




$f(t) = \frac{t}{20+t}$




$f(t) = \frac{t}{200+t}$

If $\tau$ is fixed, but there are two patches, one with $k_1$ and one with $k_2$. The bear must stay in patch with the bigger $k_i$ longer, to optimize the average rate of energy gain.

# Answers

1. A
2. D
3. C
4. D
5. A
6. B