

# Optimization: Kepler's wedding

Math 102 Section 102

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# Announcements

- ▶ Midterm: Oct 25, with Q&A session on Oct 22
  - ▶ Read the “Midterm Information” page on Canvas
  - ▶ Put your work in
- ▶ Final exam: Dec 4, 3:30 pm
  - ▶ Exam hardship: 3 exams within 24 hours
  - ▶ Exam conflict: 2 exams at the same time
  - ▶ In either case: contact Margaret Ness [jness@math.ubc.ca](mailto:jness@math.ubc.ca)

## Last time

- ▶ Global/absolute extrema
- ▶ Unconstrained optimization: cell division and logistic growth
- ▶ Constrained optimization: baculovirus
  
- ▶ Today: Kepler's wedding

# Kepler's wedding: the wine barrel problem

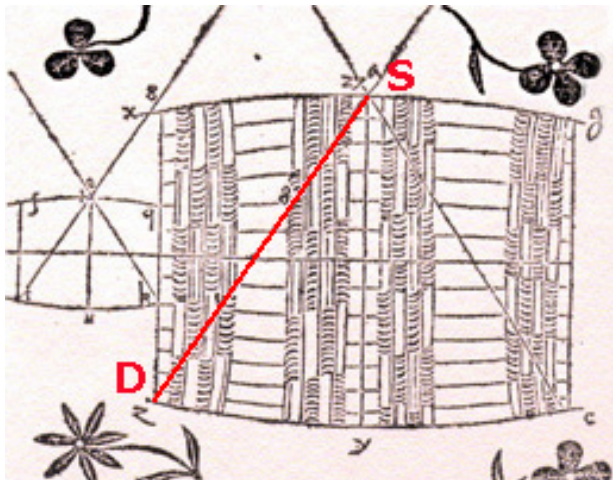


- ▶ In 1613, Kepler was going to get married.
- ▶ He wanted to buy a barrel of wine, but was intrigued by the pricing.
- ▶ He even wrote a book about it. (*Nova stereometria doliorum vinariorum* (*New solid geometry of wine barrels*))

# Kepler's wedding: the wine barrel problem

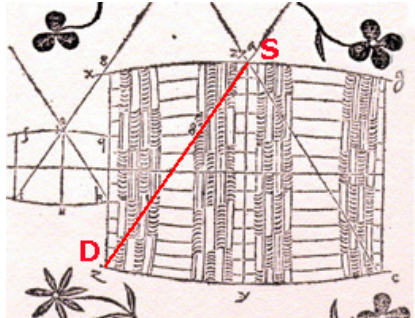


# Kepler's wedding: the wine barrel problem



# Assumptions

- ▶ The wine barrel is a cylinder.
- ▶ It is always full.
- ▶ The tap hole is right in the middle on the outer surface.



## Kepler's wedding: the wine barrel problem

Q1. With a fixed length of the wet part of the stick, does the barrel always contain the same amount of wine regardless of shape?

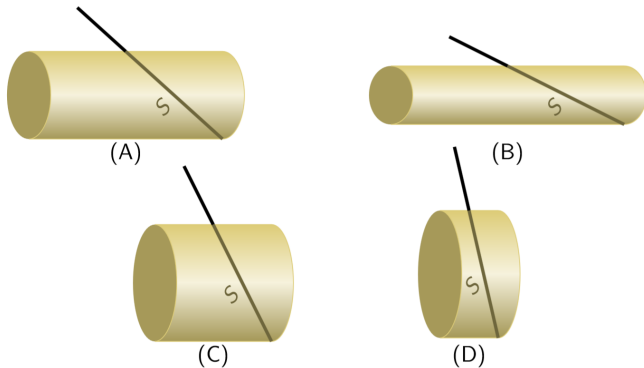
A. Yes.

B. No.

C. I don't know.



# Kepler's wedding: the wine barrel problem



There must be a shape of cylinder that contains the most amount of wine at a given  $S$ !

## Kepler's wedding: the wine barrel problem

What's the best shape? Can you formulate a mathematical model to solve this problem? (A: I have an idea; B: I have no idea.)

Tips:

- ▶ Can you formulate an optimization problem?
- ▶ Sketch a diagram of the barrel. How many quantities do you need to characterise the shape of a cylinder? What are they?
- ▶ Feel free to designate variables to represent quantities you want to use.

## Two possible formulations

- I. Given fixed  $S$ , what  $r$  and  $h$  give the barrel with the maximum volume?
- II. Given fixed volume of barrel, what  $r$  and  $h$  give the minimum  $S$ ?

Doc cam work: will be uploaded.

# Summary

To solve a constrained optimization problem:

- ▶ Identify the objective function and constraints.
- ▶ Use the constraints to eliminate extra variables in order to write the objective function in term of only one independent variable.
- ▶ Use calculus to find extrema.

# Answers

1. B