Sketching functions using calculus tools

Math 102 Section 102
Mingfeng Qiu

Oct. 3, 2018

Exercise

Q1. Inflection points are extrema of the first derivative.

A. True

B. False

Example 1

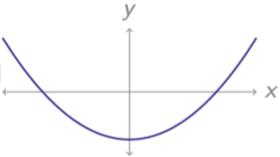
Each of the following functions have a critical point at $x = 0$.
Match the derivatives with their graphs.

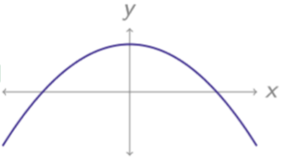
(a) $f'(x)$ negative when $x < 0$
$f'(x)$ negative when $x > 0$

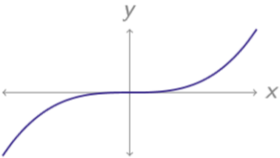
(b) $f'(x)$ positive when $x < 0$
$f'(x)$ positive when $x > 0$

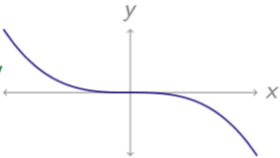
(c) $f'(x)$ negative when $x < 0$
$f'(x)$ positive when $x > 0$

(d) $f'(x)$ positive when $x < 0$
$f'(x)$ negative when $x > 0$

I


II


III


IV


Example 1: solution

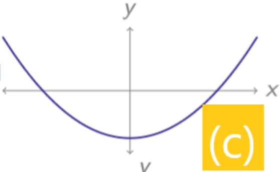
Each of the following functions have a critical point at $x = 0$.
Match the derivatives with their graphs.

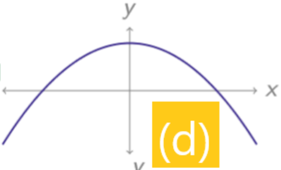
(a) $f'(x)$ negative when $x < 0$
$f'(x)$ negative when $x > 0$

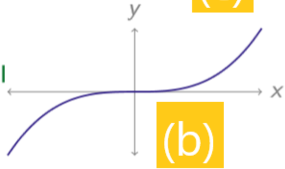
(b) $f'(x)$ positive when $x < 0$
$f'(x)$ positive when $x > 0$

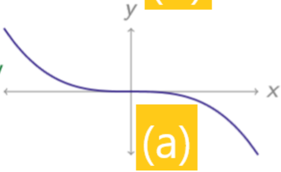
(c) $f'(x)$ negative when $x < 0$
$f'(x)$ positive when $x > 0$

(d) $f'(x)$ positive when $x < 0$
$f'(x)$ negative when $x > 0$

I


II


III


IV


Example 2

Match the second derivatives with their graphs.

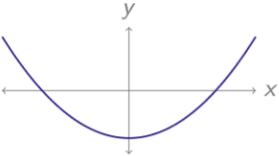
(a) $f''(x)$ negative

(b) $f''(x)$ positive

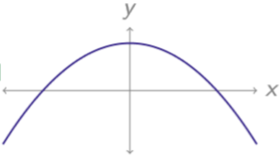
(c) $f''(x)$ negative when $x < 0$
$f''(x)$ positive when $x > 0$

(d) $f''(x)$ positive when $x < 0$
$f''(x)$ negative when $x > 0$

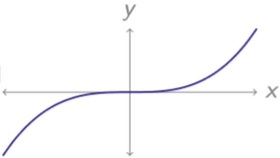
I



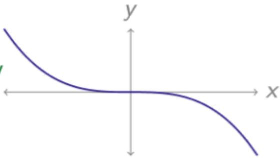
II



III



IV



Example 2: solution

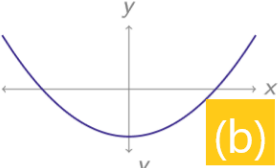
Match the second derivatives with their graphs.

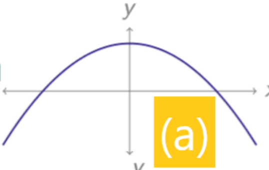
(a) $f''(x)$ negative

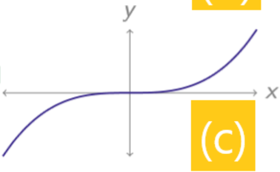
(b) $f''(x)$ positive

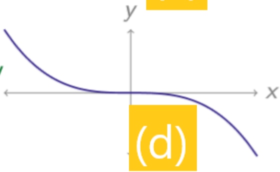
(c) $f''(x)$ negative when $x < 0$
$f''(x)$ positive when $x > 0$

(d) $f''(x)$ positive when $x < 0$
$f''(x)$ negative when $x > 0$

I


II


III


IV


Example 3

Suppose $x = a$ is a critical point of the function $f(x)$. Match the following statements.

a. $f'(x)$ changes from $-$ to $+$ at a
b. $f'(x)$ changes from $+$ to $-$ at a
c. $f''(a) = 0$
d. $f''(x)$ changes from $-$ to $+$ at a
e. $f''(x)$ changes from $+$ to $-$ at a
f. $f''(a) > 0$ and $f'(a) = 0$
g. $f''(a) < 0$ and $f'(a) = 0$

i. inflection point
ii. local max
iii. local min
iv. not a local extremum
v. could be local max, local min or neither

Example 3

Solution:

a. iii
b. iii
c. v
d. i
e. i
f. iii
g. ii

Example 4

Sketch the function

$$f(x) = \frac{1}{4}x^4 - \frac{1}{4}x^3 - 3x^2$$

- Step 0: asymptotics
- Step 1: identify zeros
- Step 2: first derivative: identify CPs
- Step 3: second derivative: identify potential IPs
- Step 4: make a table: classify all the special points and characterize the shape of the function
- Step 5: sketch

Example 5

Sketch the function

$$f(x) = \frac{(x-1)^2}{x^3}$$

- Step 0: asymptotics and discontinuities
- Step 1: identify zeros
- Step 2: first derivative: identify CPs
- Step 3: second derivative: identify potential IPs
- Step 4: make a table: classify all the special points and characterize the shape of the function
- Step 5: sketch

Answers

Q1. True

Related Exam Problems

1. Sketch the graph of the following function using calculus

$$Q(x) = \frac{x^2}{4} + \frac{2}{x}.$$