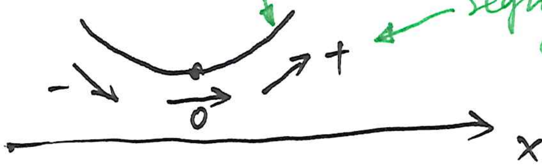
First and second derivative tests illustration

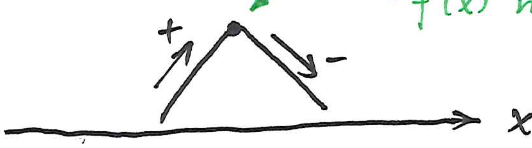
Q4.



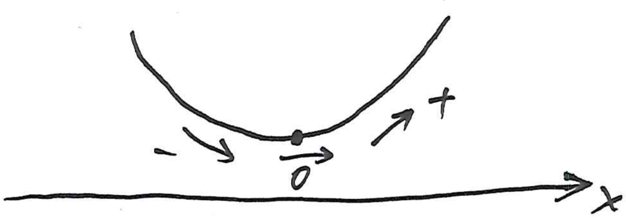
Q5. (FDT)

1) If $f'(x)$ changes from $-$ to $+$: similar to Q4

2) If $f'(x)$ changes from $+$ to $-$:



Q6.



$f''(a) > 0 \Rightarrow f'(x)$ is increasing ~~at~~ near a.

$\Rightarrow f'(x) < f'(a) = 0$, if $x < a$

$f'(x) > f'(a) = 0$, if $x > a$

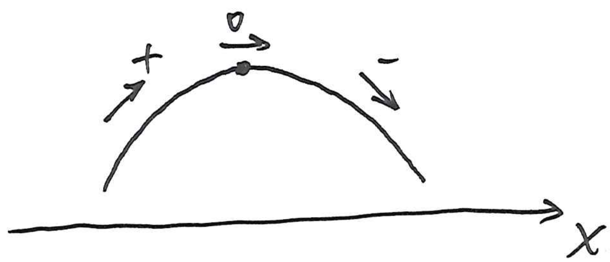
By FDT, $x=a$ is a <u>local minimum</u>.

Q7. 1) If $f''(a) > 0$: same as Q6.

(SDT)

$\Rightarrow x = a$ is a <u>local minimum</u>.

2) If $f''(a) < 0$:



$f''(a) < 0 \Rightarrow f'(x)$ is decreasing near a

$\Rightarrow f'(x) > f'(a) = 0$, if $x < a$

$f'(x) < f'(a) = 0$, if $x > a$

By FDT, $x = a$ is a <u>local maximum</u>.

Note:

1. To definitely identify extrema using FDT/SDT, one needs information of $f'(x)$ in <u>a neighborhood of</u> $x = a$, not only at a.

2. FDT/SDT are <u>sufficient but not necessary</u> conditions for local extrema. That means if $x = a$ does not pass the FDT/SDT, we can say nothing about if $x = a$ is an extremum or not.