

Newton's method: calculation demo

$$f(x) = x^3 - 2x + 1$$

$$x_0 = 0$$

$$f'(x) = 3x^2 - 2$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0 - \frac{f(0)}{f'(0)} = 0 - \frac{1}{-2} = \frac{1}{2} = 0.5$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = \frac{1}{2} - \frac{f(\frac{1}{2})}{f'(\frac{1}{2})} = \frac{1}{2} - \frac{\frac{1}{8}}{\frac{-5}{4}} = \frac{3}{5} = 0.6$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = \frac{3}{5} - \frac{f(\frac{3}{5})}{f'(\frac{3}{5})} = \frac{3}{5} - \frac{\frac{2}{125}}{-\frac{23}{25}} = \frac{71}{115} \approx 0.617$$

True root:

$$f(x) = (x-1)(x^2+x-1)$$

$$\text{one root } x = \frac{-1 + \sqrt{5}}{2} \approx 0.618$$