Worksheet: Differentiation Rules

Solution

Math 102 Section 102

Find the derivatives of the following functions

1. $f(x) = (x-8)(x^2+1)$. (Product rule)

$$f'(x) = \left(\frac{d}{dx}(x-8)\right)(x^2+1) + (x-8)\frac{d}{dx}(x^2+1)$$

$$= 1\cdot(x^2+1) + (x-8)\cdot 2x$$

$$= x^2+1+2x^2-16x = 3x^2-16x+1$$

2. $f(x) = \frac{x^2-9}{x^2+9}$.

Method ①: chain rule + power rule

$$f(x) = 1 - \frac{18}{x^2+9}$$

$$f'(x) = 0 - (-1)\frac{18\cdot 2x}{(x^2+9)^2} = \frac{36x}{(x^2+9)^2}$$

Method ②: quotient rule

$$f'(x) = \frac{2x(x^2+9) - (x^2-9)\cdot 2x}{(x^2+9)^2}$$

$$= \frac{36x}{(x^2+9)^2}$$

3. $f(x) = 5(x^2-3x)^{10}$. (Chain rule)

$$f'(x) = 50(x^2-3x)^9(2x-3)$$

4. $f(x) = \frac{x}{\sqrt{x^2+d^2}}$. (Quotient rule)

$$f'(x) = \frac{1\cdot\sqrt{x^2+d^2} - x\cdot\frac{1}{2}(x^2+d^2)^{-\frac{1}{2}}\cdot 2x}{x^2+d^2}$$

$$= \frac{x^2+d^2-x^2}{(x^2+d^2)^{3/2}} = \frac{d^2}{(x^2+d^2)^{3/2}}$$

Antiderivatives.

1. If $f'(x) = mx+b$, find $f(x)$.

$f(x) = \frac{1}{2}mx^2 + bx + C$ for some constant C.

2. If $g'(t) = 5(t^3+t^2)^4(3t^2+2t)$, find $g(t)$.

$g(t) = (t^3+t^2)^5 + A$ for some constant A.

key idea:

notice $\frac{d}{dt}(t^3+t^2) = 3t^2+2t$.