The derivative: more examples and a computational view

Math 102 Section 102
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Sep. 19, 2018

Due this week

- Sep 19 (today): WeBWork Pre-lecture 3.2
- Sep 20 (Tomorrow): WeBWork Assignment 2
- Due time: 9:00 pm instead of 11:59 pm

Announcements

- Office Hours:
  - Tue: 2:30-4:00
  - Wed: 3:00-4:00
  - Location: LSK 300B (next to MLC)
- This time, did you do the problems in the slides?
  A. I was working too hard on the practice problems and did not notice any joke.
  B. The joke was so terrible! You just gave me this ~~crap~~ after my hard work on the problems?!!
  C. I read the ~~horrible~~ joke but did not do the practice problems.
  D. What the hell is going on? Am I in the right classroom?

Last time

- Derivative: how fast a function is changing
- Tangent line: zoom in
- Continuity and Differentiability
- Sketch the derivative of a given function

Today

- More examples on derivatives
- Compute numerical approximations of a derivative

Derivative

Definition (Derivative)

The derivative of a function $y = f(x)$ at x_0 is

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}.$$

- We can also write $\left.\frac{dy}{dx}\right|_{x_0}$ to denote $f'(x_0)$.
- and write $f'(x)$ or $\frac{dy}{dx}$ to denote the derivative as a function of x.

Derivative

Example

Derivative of $f(x) = \frac{1}{x}$

$$
\begin{aligned}
f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \to 0} \frac{1}{h} \left(\frac{1}{x+h} - \frac{1}{x} \right) \\
&= \lim_{h \to 0} \frac{1}{h} \left(\frac{x - (x+h)}{(x+h)x} \right) \\
&= \lim_{h \to 0} \frac{-1}{(x+h)x} \\
&= \frac{1}{x^2}
\end{aligned}
$$

Derivative

Example

Derivative of $f(x) = \frac{1}{\sqrt{x}}$

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h\to 0} \frac{1}{h}\left(\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}\right) \\
&= \lim_{h\to 0} \frac{1}{h}\frac{\sqrt{x} - \sqrt{x+h}}{\sqrt{x}\sqrt{x+h}} \\
&= \lim_{h\to 0} \frac{1}{h}\frac{x - (x+h)}{\sqrt{x}\sqrt{x+h}(\sqrt{x} + \sqrt{x+h})} \\
&= -\frac{1}{\sqrt{x}\sqrt{x+h}(\sqrt{x} + \sqrt{x+h})} \\
&= -\frac{1}{2(\sqrt{x})^3}
\end{aligned}$$

Derivative: sketching

Example

Let $f(x) = \frac{2x^2}{1+x^2}$. Sketch the graph of the function and its derivative.

Check using Desmos.com:

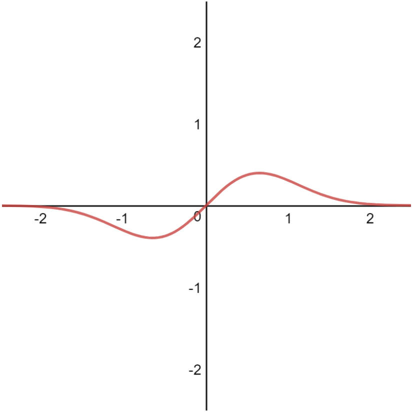
https://www.desmos.com/calculator/zynqzycx5y

Notice that $f(x)$ is an even function. The derivative of any even function is an odd function. Why? After you learn the chain rule...

Derivative: sketching

Example

Sketch the first derivative of $f(x) = e^{-x^2}\sin(x)$.

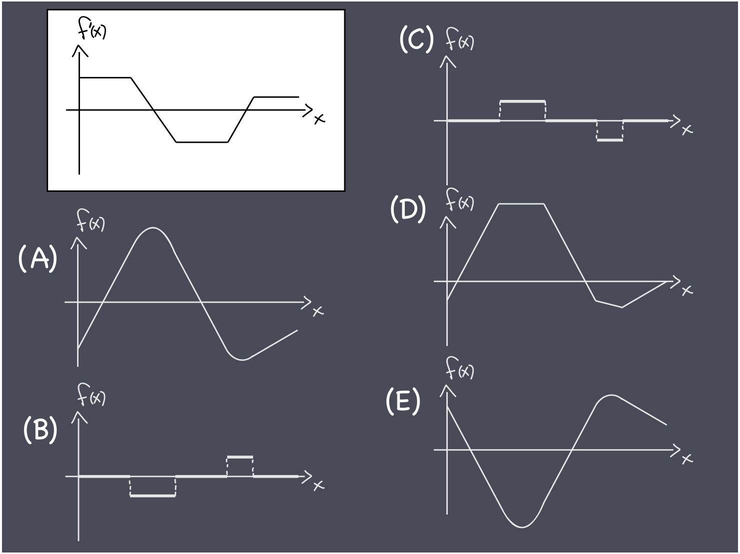


See if your graph looks right:

https://www.desmos.com/calculator/gparyguvgc

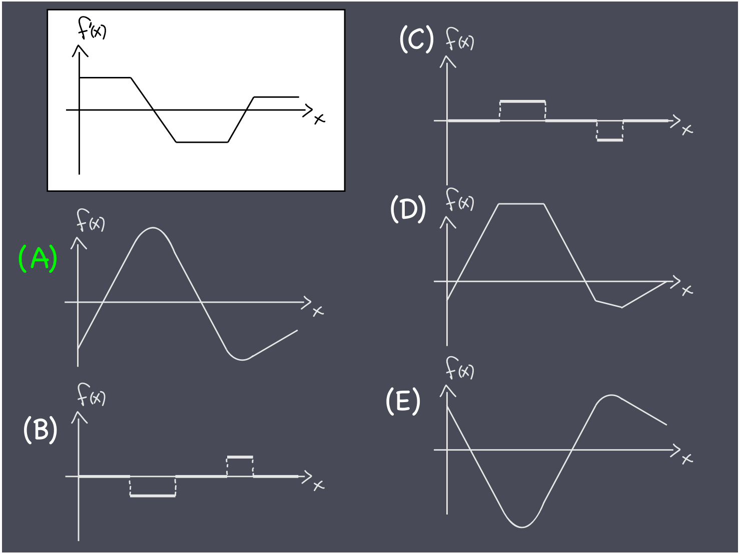
Going backwards

Q1. Given $f'(x)$, sketch the original function.



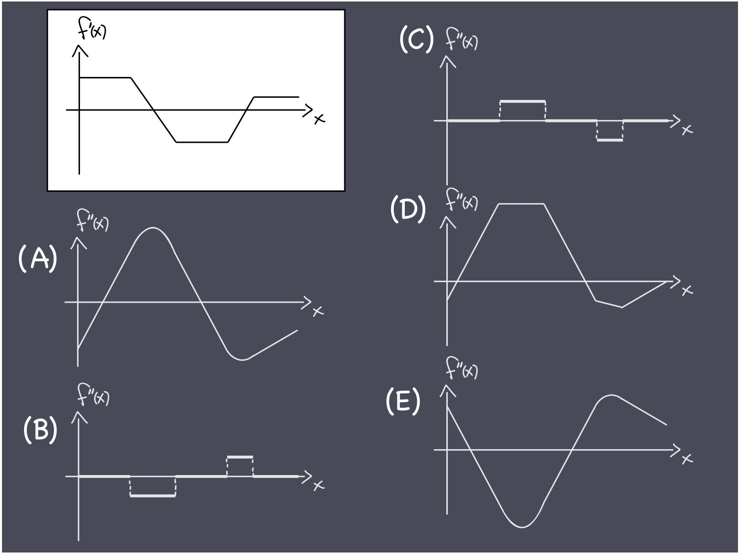
Going backwards

Q1. Given $f'(x)$, sketch the original function.



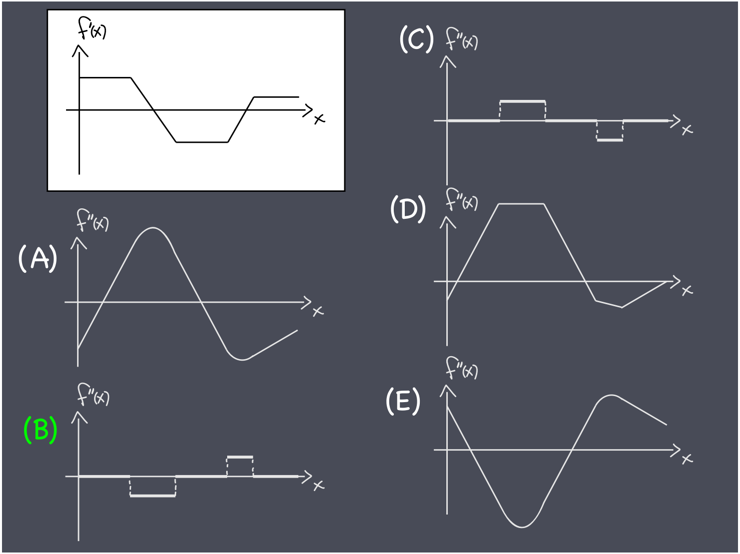
Second derivative

Q2. Given $f'(x)$, sketch the second derivative, $f''(x)$.



Second derivative

Q2. Given $f'(x)$, sketch the second derivative, $f''(x)$.



Derivative: computational aspects

Spreadsheets?

- Useful common tool:
  - Budgets
  - Lab data
  - Grades
- Used in business, engineering, accounting
- Big data and scientific computing
- Same ideas as more abstract programming
- Enhance understanding on calculus
  - Approximations
  - Accuracy and refining a result
  - Iteration methods

Approximating the derivative

Q3. The derivative of a function $f(t)$ can be approximated by

A. $f'(t) \approx \frac{f(t+h)-f(t)}{h}$ for a large value of h

B. $f'(t) \approx \frac{f(t+h)-f(t)}{h}$ for a small value of h

C. $f'(t) \approx \frac{f(t+h)-f(t)}{t}$ for a large value of h

D. $f'(t) \approx \frac{f(t+h)-f(t)}{t}$ for a small value of h

Geometric interpretation: use a secant line to approximate the tangent line.

Approximating the derivative

Our goals:

- Use a spreadsheet to compute an approximation to the derivative of $f(x) = x^3$ over the interval $0 \leq x \leq 1$, using different Δx.
- Compare to the exact derivative.

Mingfeng's example:
https://docs.google.com/spreadsheets/d/11l0hj_k5IBrj03ayFXtN4Q4khKs0l1996tR5L8v0h6g/edit?usp=sharing

Position, velocity, & acceleration

- $x(t) =$ position
- $v(t) = x'(t) =$ velocity
- $a(t) = v'(t) = x''(t)$ acceleration

Acceleration is the second derivative of position.

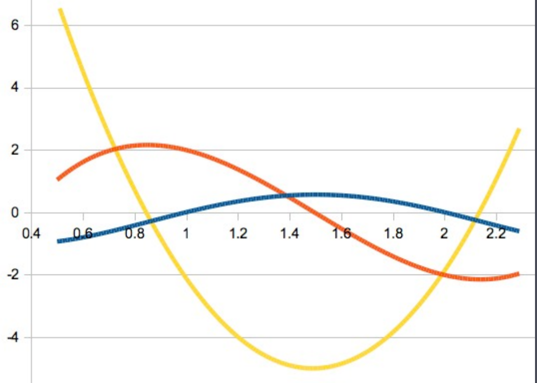
Which is x, v, a?

(A) x, v, a

(B) x, v, a

(C) x, v, a

(D) x, v, a



Check max/mins --> zeros, check inc/dec --> +/-.

Q4

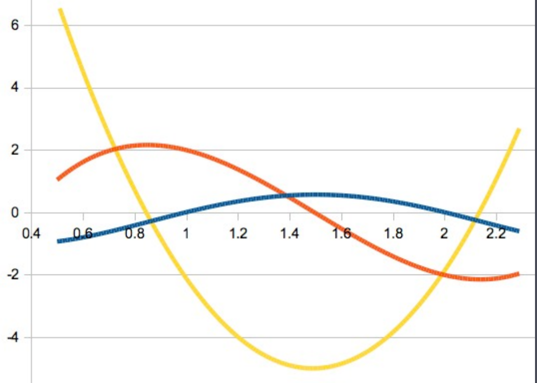
Which is x, v, a?

(A) x, v, a

(B) x, v, a

(C) x, v, a

(D) x, v, a



Check max/mins --> zeros, check inc/dec --> +/-.

Today...

- Algebraic skills in computing limits and derivatives
- More sketching examples. Key ideas:
  - Maxima/minima correspond to zeros in derivative
  - Signs of slope
- Going backwards: given $f'(x)$, sketch $f(x)$.
- Position, velocity and acceleration and the second derivative
- Approximating the derivative computationally (an introduction to scientific computing)

Answers

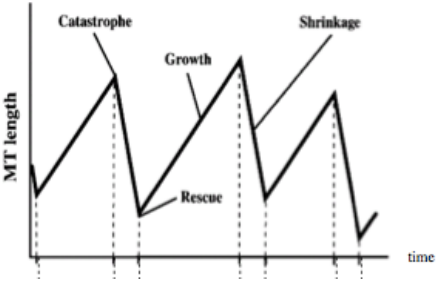
1. A
2. B
3. B
4. C

Related exam problems

1. When we approximate the derivative of a function by $f'(t) \approx \frac{f(t+h)-f(t)}{h}$ then we are
   A. Approximating the slope of the secant line by the slope of a tangent line.
   B. Approximating the slope of the tangent line by the slope of a secant line.
   C. Approximating the slope of the tangent line by the value of the function.
   D. Trying to avoid having to calculate a limit.
   E. None of the above.

Related exam problems

(a) Microtubules (MTs) are biological polymers important in cell structure, cell division, and transport of material inside cells. The length of microtubules (MT length) grows and shrinks as shown in the following figure from Janulevicius *et al.* (2009) *Biophys. J.* 90:788-798. Use this figure to draw a sketch of MT growth rate (i.e. rate of change of microtubule length per unit time) over the same time interval.
(b) Which has a greater magnitude: the rate of shrinkage (per unit time) or the rate of growth (per unit time)?



The fun page[1]

The difference between an introvert and extrovert mathematicians is: An introvert mathematician looks at his shoes while talking to you. An extrovert mathematician looks at your shoes.

[1]Oh my god! Now I feel inevitably obliged to give you another joke. You see, as a person not endowed with the humor as I would have hoped to be, this becomes a real challenge!