Hill functions
Math 102 Section 102

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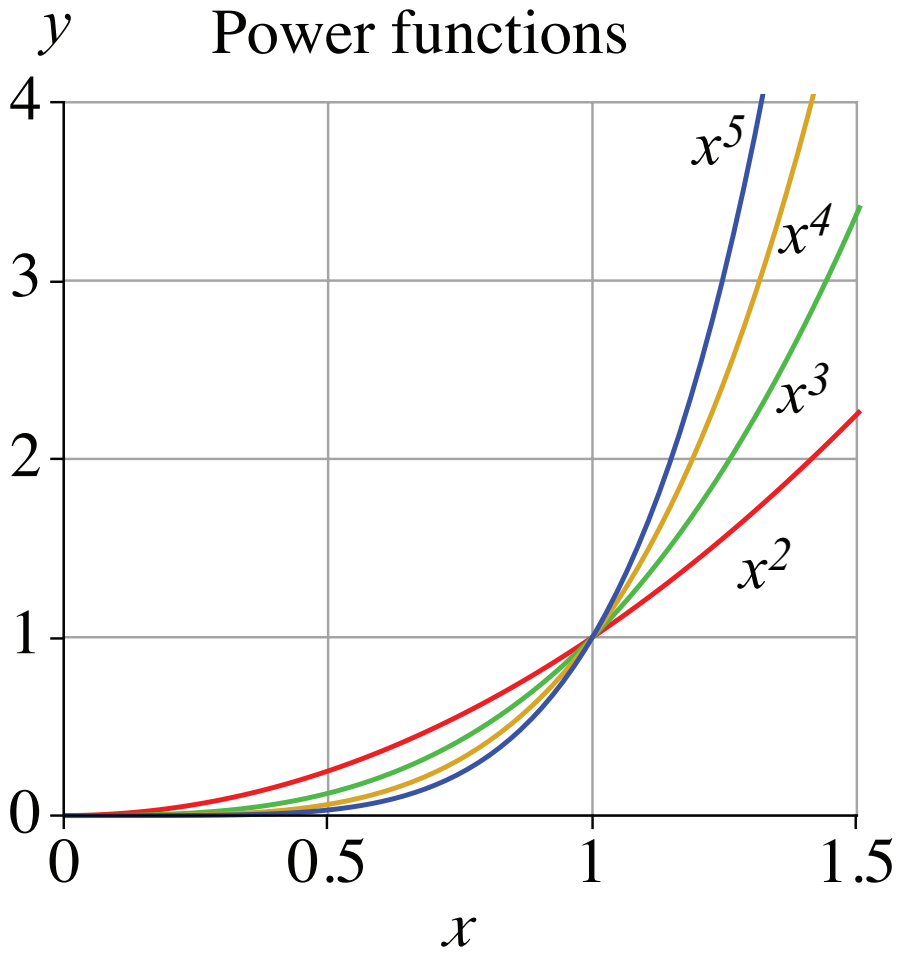
Due this week

- Today: WW logstics (anyone not knowing how to log in?)
- Tue: WW pre-lecture 2.1
- Wed: OSH 0
- Thu: WW pre-lecture 2.2
- Fri: OSH 1 (start early)
- Sun: WW diagnostic (1 hour, without resources)

Last time: power functions

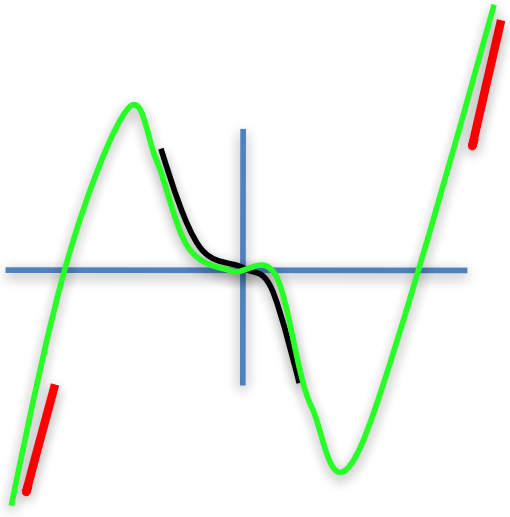
- Small powers dominate close to $x = 0$; large powers dominate for large x.



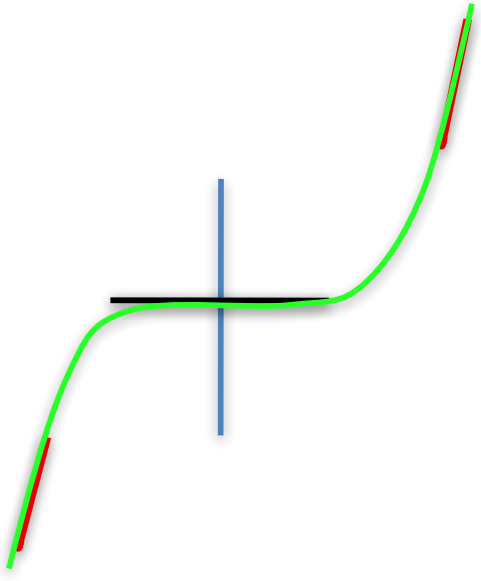
Last time: sketching simple polynomials

Example (Sketch $y = x^5 + ax^3$.)

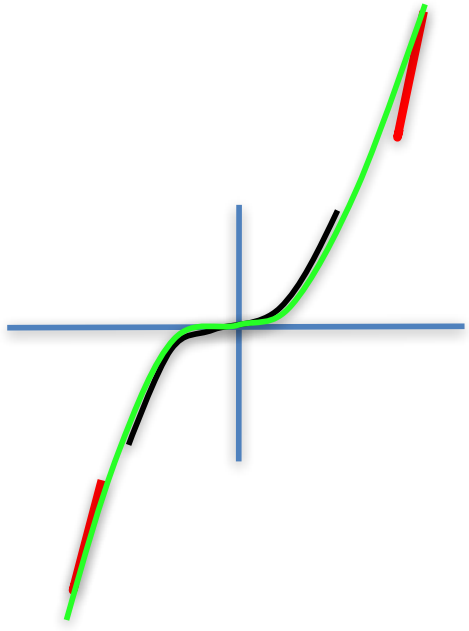
$a < 0$:



$a = 0$:



$a > 0$:



Correction: number of zeros

Today

- Rational functions and Hill functions
- Sketch a Hill function
- Michaelis-Menten model in biochemistry

Rational functions

- A rational function is a function that can be written as

$$y = \frac{p_1(x)}{p_2(x)},$$

where $p_1(x)$ and $p_2(x)$ are polynomials.

Example (Hill function)

Draw a sketch of

$$y = \frac{Ax^n}{a^n + x^n}$$

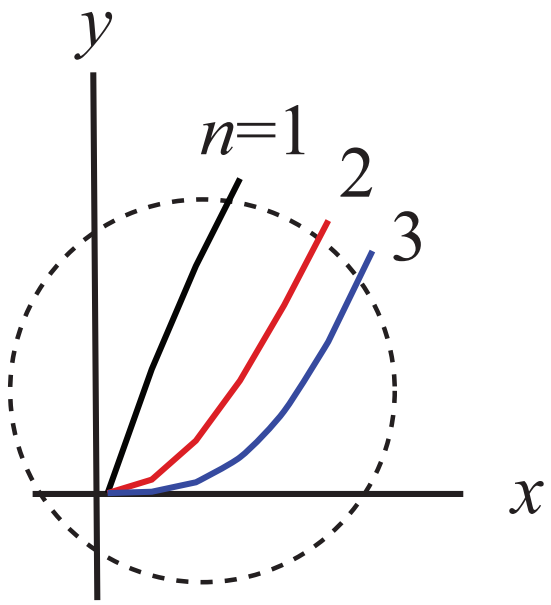
for $x \geq 0$. $(A, a > 0, n \geq 1)$

Rational functions

$$y = \frac{Ax^n}{a^n + x^n}, \ x \geq 0.$$

- $x \ll a$ (much smaller than a):

$$a^n + x^n \approx a^n \Rightarrow y \approx \frac{Ax^n}{a^n} = \frac{A}{a^n}x^n.$$

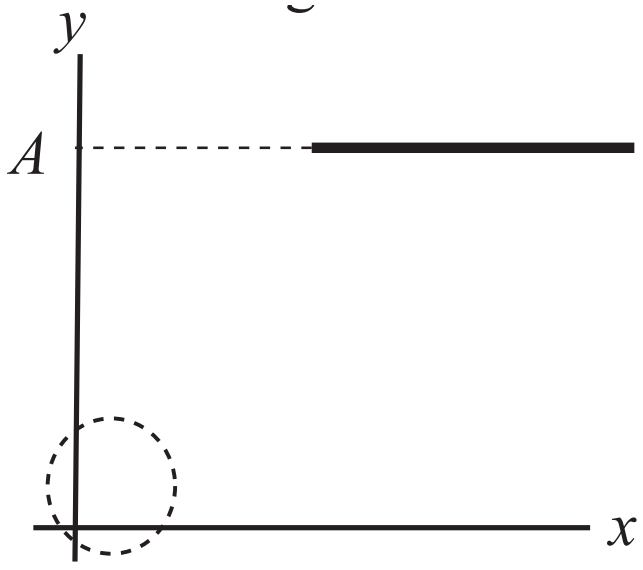


Rational functions

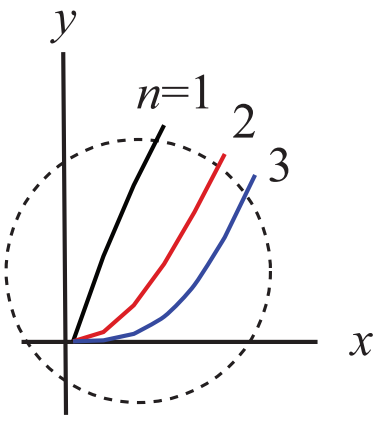
$$y = \frac{Ax^n}{a^n + x^n}, \ x \geq 0.$$

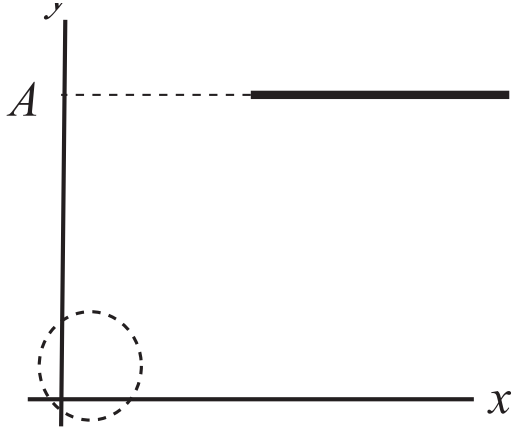
- $x \gg a$ (much bigger than a):

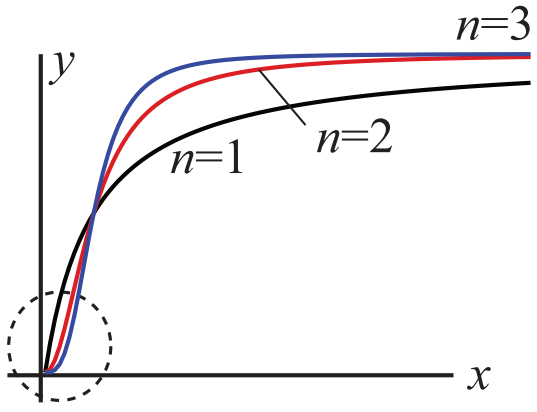
$$a^n + x^n \approx x^n \Rightarrow y \approx \frac{Ax^n}{x^n} = A$$



Hill function: $y = \frac{Ax^n}{a^n + x^n}, \; x \geq 0$







Hill function: $y = \frac{Ax^n}{a^n + x^n}, \ x \geq 0$

Q1. The asymptote for the Hill function is

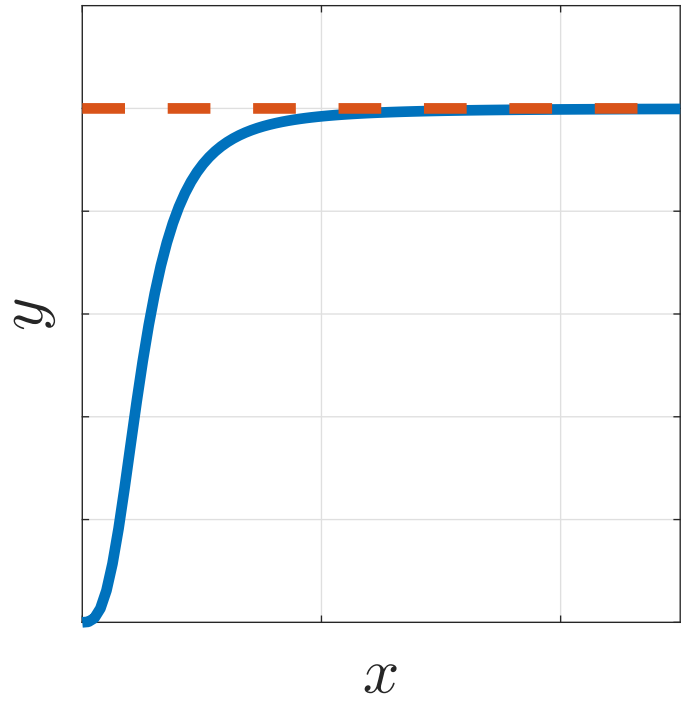
A. A

B. $A/2$

C. a

D. $a/2$

E. a^n



- $y = A$ is the maximal response
- We also say

$$\lim_{x \to \infty} \frac{Ax^n}{a^n + x^n} = A.$$

Hill function: $y = \frac{Ax^n}{a^n + x^n}, \; x \geq 0$

Q2. The value of x for half-maximal response is

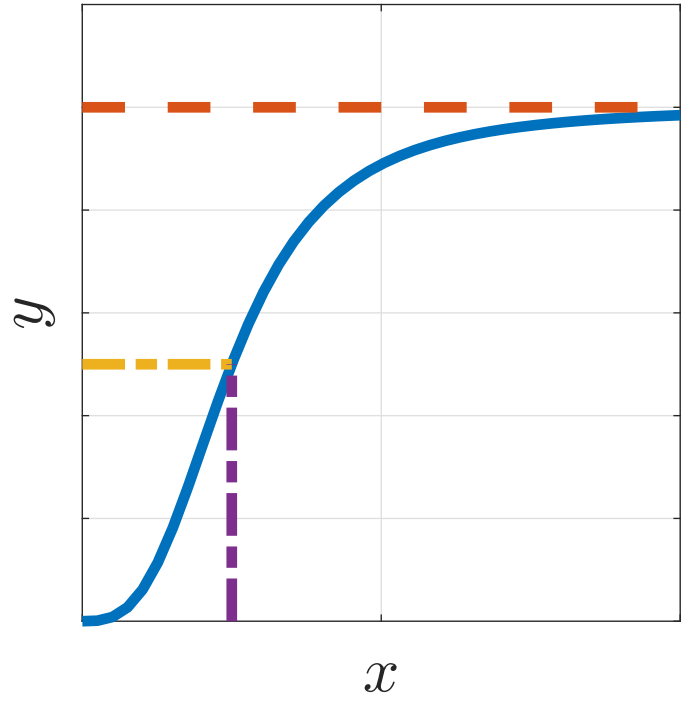
A. A

B. $A/2$

C. a

D. $a/2$

E. a^n



- $x = a$ is the half-max

Hill function: $y = \frac{Ax^n}{a^n+x^n},\ x \geq 0$

Definition (Hill function)

A rational function which has the form of

$$y = \frac{Ax^n}{a^n + x^n},\ x \geq 0,$$

where $A, a > 0$ and n is a non-negative integer, is called a Hill function. A is the horizontal aymptote, n the coefficient, and a the half-max.

Hill function: $y = \frac{Ax^n}{a^n + x^n}, \ x \geq 0$

Q3. Why is it called a Hill function?

A. Because it looks like a hill

B. Because it describes an increasing function

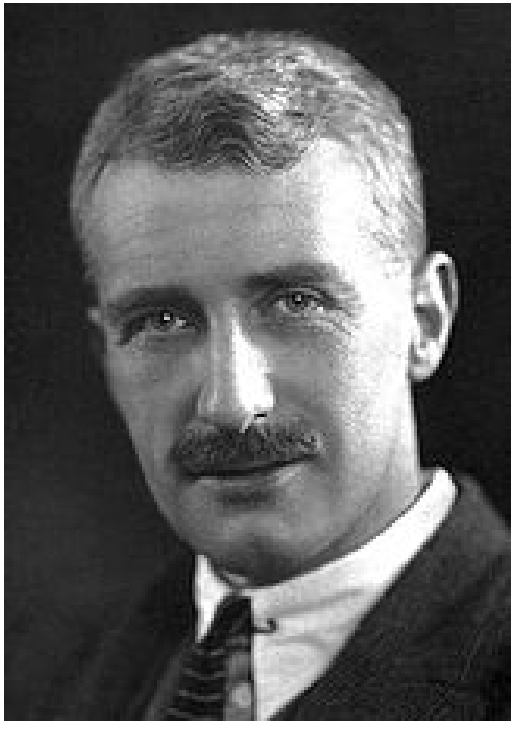
C. Because it was named after A.V. Hill

D. Because in biology it describes a Hill process

Why is it called a Hill function?

Hill functions are named after, Archibald Hill, a Nobel Prize winning muscle physiologist.

The Combinations of Haemoglobin with Oxygen and with Carbon Monoxide. I *Biochem. J* 1913 Oct; 7(5): 471-480.
https://www.ncbi.nlm.nih.gov/pmc/articles/PMC1550542/

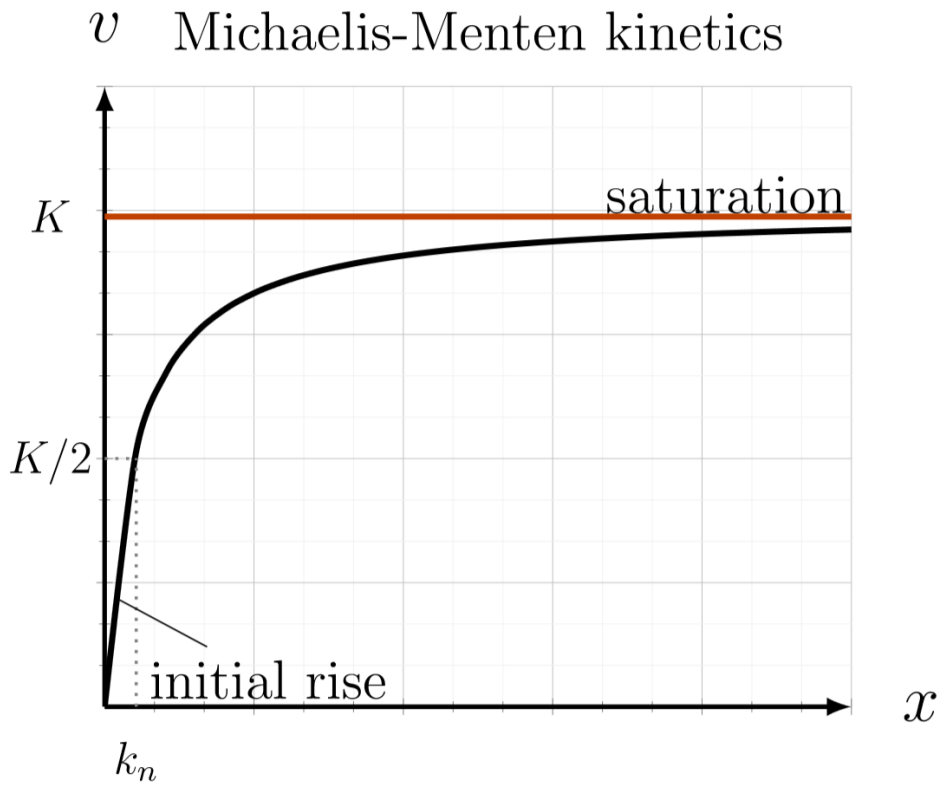


https://en.wikipedia.org/wiki/Archibald_Hill

Speed of an enzyme reaction

Michaelis-Menten kinetics:

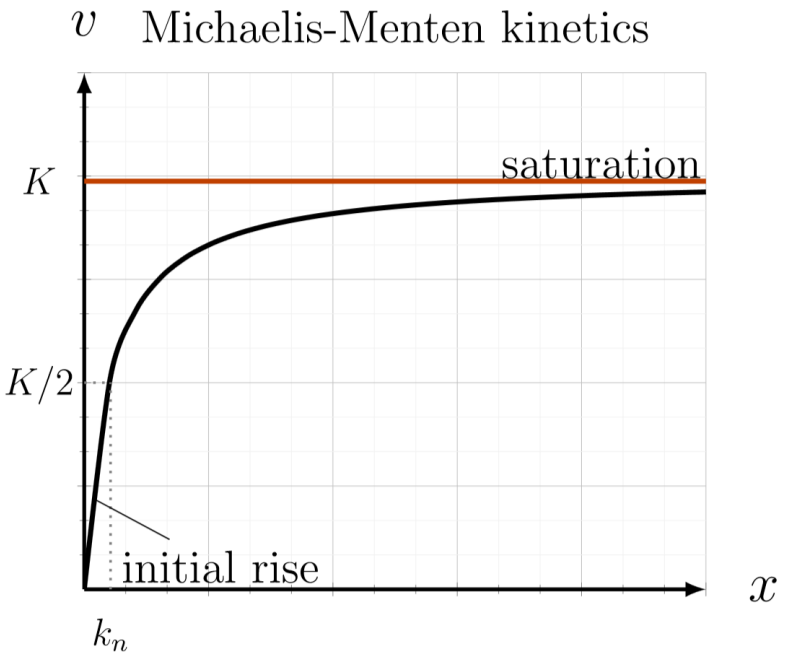
$$E + S \rightleftharpoons C \rightarrow E + P$$



Speed of an enzyme reaction

Speed of reaction

$$v = \frac{Kx}{k_n + x}$$



Today

- Rational functions
- Hill functions (horizontal asymptote, half-max, coefficient)
- Sketch a Hill function
- Enzyme reaction speed can be modelled using Hill functions

Answers

1. A
2. C
3. C