Derivatives of Trig and Inverse Trig Functions

Math 102 Section 102 Mingfeng Qiu

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I'm planning to have additional office hours next week. Next Monday (Dec 3), which time works best for you?

- A. 1:00-2:00 (most popular)
- B. 2:00-3:00 (next popular)
- C. 4:00-5:00
- D. 5:00-6:00

- Trigonometric functions can be used to describe rhythmic processes. To do that, one just needs to figure out the amplitude, frequency and phase shift.
- To be invertible, trig functions have to be restricted to certain domains.
- Triangles and identities are useful when simplifying inverse trigonometric relationships.

- Define and calculate derivatives of trig functions
- and those of their inverses
- Use trig functions in related rates problems

Derivatives of trig functions

Derivative of cosine and sine

Cosine:

$$\frac{d}{dt}\cos t = -\sin t$$
$$\frac{d}{dt}\sin t = \cos t$$

Sine:

The special relationship between these functions means that they satisfy the following equations:

$$\frac{dx}{dt} = -y$$
$$\frac{dy}{dt} = x$$

Derivative of cosine and sine: visualization

- Recall: the derivative of displacement in time = velocity
- Recall: trig functions are closely related to motion along a circle
- Desmos demo

Derivative of $\sin x$

$$\frac{d\sin(x)}{dx} = \lim_{h \to 0} \frac{\sin(x+h) - \sin(x)}{h}$$
$$= \lim_{h \to 0} \frac{\sin(x)\cos(h) + \sin(h)\cos(x) - \sin(x)}{h}$$
$$= \lim_{h \to 0} \left(\sin(x)\frac{\cos(h) - 1}{h} + \cos(x)\frac{\sin(h)}{h}\right)$$
$$= \sin(x)\left(\lim_{h \to 0} \frac{\cos(h) - 1}{h}\right) + \cos(x)\left(\lim_{h \to 0} \frac{\sin(h)}{h}\right)$$
$$= \cos(x)$$

Second derivative of $y(t) = \sin(t)$?

- Q1. $y = \sin(t)$ is a solution to the following differential equation. A. $\frac{d^2y}{dt^2} = -y$ B. $\frac{d^2y}{dt^2} = y$ C. $\frac{dy^2}{dt} = y$ D. $\frac{d^2y}{dt^2} = -t$
 - E. Send help

Second derivatives

$$\frac{d^2 \sin t}{dt^2} = -\sin t \Rightarrow \frac{d^2 y}{dt^2} = -y$$
$$\frac{d^2 \cos t}{dt^2} = -\cos t \Rightarrow \frac{d^2 x}{dt^2} = -x$$

The trig functions sin t and cos t are both solutions to the differential equation

$$\frac{d^2y}{dt^2} = -y$$

- Q2. The derivative of $\tan x$ is
 - A. $\cot x$
 - $\mathsf{B.} \cot x$
 - C. $\cos^2(x)$
 - D. $\sec^2(x)$
 - E. Send help

Use quotient rule.

Derivatives of inverse trig functions

Derivatives of inverse trig functions

- Q3. If $y = \arcsin x$, with $-1 \le x \le 1$, what is $\frac{dy}{dx}$?
 - A. $\arccos x$
 - **B**. $\arctan x$

C.
$$\sqrt{1-x^2}$$

D.
$$\frac{1}{\sqrt{1-x^2}}$$

E. $\frac{1}{1+x^2}$

In order to solve this problem, it's actually easier to think about the derivative of a general inverse function $f^{-1}(x)$. And how did we find the derivative of a log function?

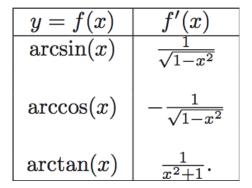
Derivatives of inverse functions

- Consider an inverse function $y = f^{-1}(x)$
- This means x = f(y)
- Take derivative w.r.t. x on both sides!

$$1 = f'(y) \cdot \frac{dy}{dx}$$
$$\frac{dy}{dx} = \frac{1}{f'(y)}$$

The derivative of the inverse function is the reciprocal of the derivative of the original function! Be careful of variables.
Don't forget to substitute y with the appropriate expression in x!

Derivatives of Inverse Trig Functions



Trig functions and related rates

Example

In a prison, a search light is 30 m from a straight wall. The search light rotates at a rate of 3 rounds per minute. Let P be the point on the wall closest to the light tower. The light beam hits the wall at some spot (with negligible cross-sectional width), and that spot moves as the search light rotates. When the light spot on the wall is 10 m away from P, how fast is the spot moving?

(Document camera)

- $\frac{d}{dx}\sin(x) = \cos(x)$, $\frac{d}{dx}\cos(x) = -\sin(x)$
- These derivatives can be demotrated with motion on a circle, and derived rigorously using the limit definition.
- Derivatives of other trig functions may be found by using differentiation rules.
- Derivatives of inverse trig functions can be derived with implicit differentiation.

1. Show that the derivative of $\arctan x$ is $\frac{1}{1+x^2}$

Answers

C
D
D

1. Use Newton's method to find the smallest positive critical point of the function

$$g(x) = e^{-x}\sin(10x)$$