Derivative of sin(x)

$$\frac{d}{dx}\sin(x) = \lim_{h\to 0}\frac{\sin(x+h)-\sin(x)}{h}$$

$$= \lim_{h\to 0}\frac{\sin x\cosh + \cos x\sin h - \sin x}{h}$$

$$= \lim_{h\to 0}\left[\frac{\sin x(\cos h - 1)}{h} + \frac{\cos x\sin h}{h}\right]$$

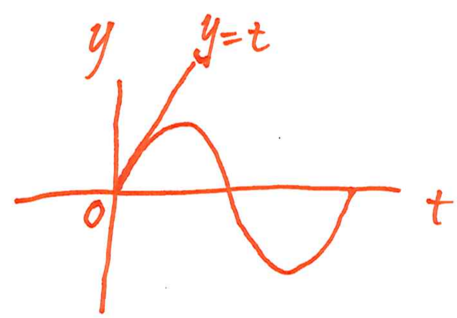
$$= \sin x\left(\lim_{h\to 0}\frac{\cos h - 1}{h}\right) + \cos x\left(\lim_{h\to 0}\frac{\sin h}{h}\right)$$

$$= \sin x \cdot 0 + \cos x \cdot 1$$

$$= \underline{\cos x}$$

Recall $\lim_{t\to 0}\frac{\cos t - 1}{t} = 0$

$\lim_{t\to 0}\frac{\sin t}{t} = 1$.



Derivative of arcsin(x)

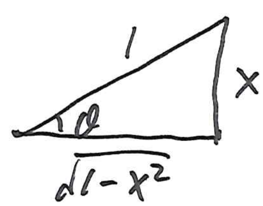
$y = \arcsin(x)$

In this case, $f = \sin$

$$\frac{dy}{dx} = \frac{1}{f'(y)} = \frac{1}{\frac{d}{dy}(\sin y)} = \frac{1}{\cos y}$$

Let $\theta = \arcsin(x)$. Then $\sin\theta = x$

$\Rightarrow \cos\theta = \sqrt{1-x^2}$



So $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{1-x^2}}$

Alternative:

$y = \arcsin(x)$

$x = \sin(y)$

Implicit differentiation w.r.t. x. ...

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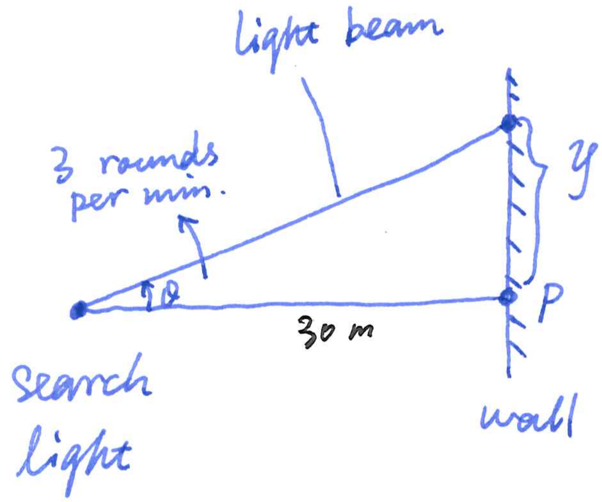


Q: when $y = 10$ (m) $\frac{dy}{dt} = ?$

Geometry: $\tan\theta = \frac{y}{30}$

$$y = 30\tan\theta$$

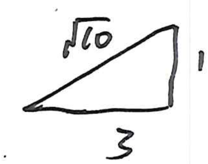
Differentiate:

$$\frac{dy}{dt} = 30 \frac{1}{\cos^2\theta} \frac{d\theta}{dt}$$

We know $\frac{d\theta}{dt} = \frac{3 \text{ rounds}}{\text{min}} = 3 \cdot 2\pi = 6\pi$ (radians/min)

At the moment $y = 10$, $\tan\theta = \frac{1}{3}$

$$\cos\theta = \frac{3}{\sqrt{10}}$$



$$\frac{dy}{dt} = 30 \cdot \frac{10}{9} \cdot 6\pi = \frac{600\pi}{3} \text{ (m/min)}$$