Trig Functions, Their Inverses and Derivatives

Math 102 Section 102
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Nov. 26, 2018

Due due due due due...

- Nov 26 (Today): Pre-lecture 13.1
- Nov 28 (Wednesday): Pre-lecture 13.2
- Nov 29 (Thursday): Assignment 12

Assignments due: 9:00 pm

- Dec 4 (Exam day), 3:30 pm: Assignment 13

Ammoucement

Final exam question sessions

- Nov 29 (Thu), 4-7 pm
- Nov 30 (Fri), 4-7 pm
- Location: SWNG 122

Today: learning goals

- Fit trig functions to describe rhythmic processes
- Define and apply inverse trig functions
- Derivatives of trig functions (moved to Wednesday)

OSH 6 - Q3

3. I Am My Own Grandpa

where by "grandpa" we mean "derivative"

3a. True or false: the function $y = e^x$ satisfies the differential equation $\frac{dy}{dx} = y$.

3b. Find a constant function $y = c$ that also satisfies the differential equation $\frac{dy}{dx} = y$.

3c. We define a function y(x) as the (infinite) sum below:

$$y(x) = 1 + \frac{x}{1} + \frac{x^2}{2 \cdot 1} + \frac{x^3}{3 \cdot 2 \cdot 1} + \frac{x^4}{4 \cdot 3 \cdot 2 \cdot 1} + \frac{x^5}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} + \cdots$$

What is $y(0)$?

3d. Does $y(x)$ (the function defined above) satisfy the differential equation $\frac{dy}{dx} = y$? Explain your reasoning.

3e. We want to generate approximate values of y(x) on the interval [-2,2]. Since we can't add up infinitely many numbers, we'll use the function $\tilde{y}$ instead of y, where $\tilde{y}$ is only the sum of the first eleven terms:

$$\tilde{y} = 1 + \frac{x}{1} + \frac{x^2}{2 \cdot 1} + \frac{x^3}{3 \cdot 2 \cdot 1} + \cdots + \frac{x^{10}}{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

Use a spreadsheet to generate the values $\tilde{y}(x)$ for x in the interval [-2,2], with x-increments of 0.1. That is: find $\tilde{y}(-2)$, $\tilde{y}(-1.9)$, $\tilde{y}(-1.8)$, etc. Give a screenshot of your work.

Hint: the denominators are factorials, usually computed in spreadsheets as =FACT(n). So, for example, =FACT(3) will give you 6, because 3*2*1 is 6.

3f. Use your spreadsheet to generate values of e^x for the same x-values you used above. Screenshot the values. What does this suggest about the function y(x)?

Application of trig functions

- Trig functions can be used to descibe some rhythmic/periodic/oscillatory processes.

Example (Fitting a trig function to a rhythmic process)

The level of a certain hormone in the bloodstream fluctuates between undetectable concentration at $t = 7:00$ and 100 ng/ml (nanograms per millilitre) at $t =$19:00 hours. Approximate the cyclic variations in this hormone level with an appropriate periodic trigonometric function. Let t represent time in hours from 0:00 hrs through the day.

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Inverse trig functions

Recall: inverse functions

Definition (Inverse function)

Given a function $y = f(x)$, its inverse function, denoted as f^{-1}, satisfies

$$f^{-1}(f(x)) = x.$$

Q1. The inverse function of $f(x)$ is the mirror image of the graph of $f(x)$ to the line $y = x$.

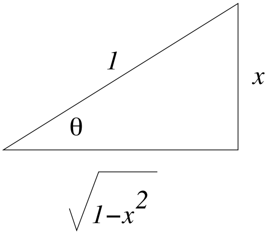
A. True.

B. False.

- Only one-to-one (or 1-1 or bijective) functions are invertible.

Inverse trig functions

- $f(x) = \sin x$
- $f^{-1}(x) = \arcsin(x)$ "the angle whose sine is x"



- Because $\sin x$ is periodic, we need to restrict the domain to define an inverse function.
- https://www.desmos.com/calculator/zx6c0htith
- Inverse trig functions are only inverses of the trig functions on their restricted domains!

Inverse trig functions

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Inverse trig functions

Q2. Simplify the expression $y = \tan(\arcsin(x))$

A. $\sqrt{1 - x^2}$

B. $\frac{x}{\sqrt{1-x^2}}$

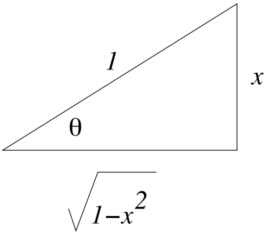
C. $\frac{1}{x}$

D. x

E. $\frac{\sqrt{1-x^2}}{x}$

Inverse trig Functions

$$y = \tan(\arcsin(x))$$



- $\theta = \arcsin(x)$
- $y = \tan\theta = \frac{x}{\sqrt{1-x^2}}$

Inverse trig Functions

Q3.

$$\arcsin\left(\sin\left(\frac{5\pi}{6}\right)\right) =$$

A. $\frac{1}{2}$

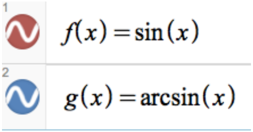
B. $\frac{\sqrt{3}}{2}$

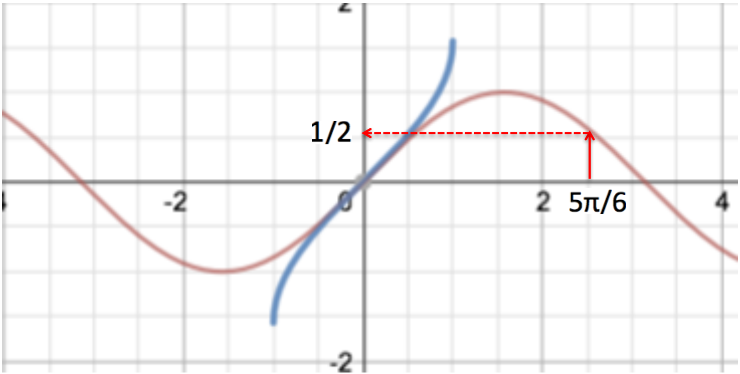
C. $\frac{\pi}{6}$

D. $\frac{5\pi}{6}$

E. $\frac{7\pi}{6}$

Sin(5π/6)=1/2

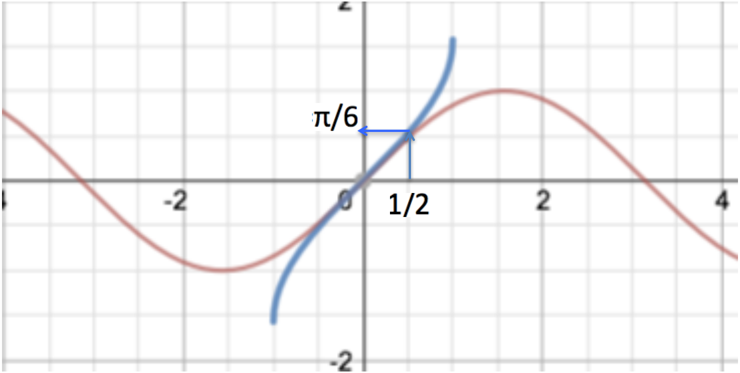




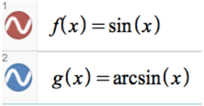
Arcsin(1/2)=π/6

1 $f(x) = \sin(x)$

2 $g(x) = \arcsin(x)$



arcsin(sin(x))≠x



In this example, we found that:

$$\arcsin(\sin(5\pi/6)) = \pi/6 \neq 5\pi/6$$

However, **<u>on the restricted domains</u>**,

$-\pi/2 \leq x \leq \pi/2$ and $-1 \leq x \leq 1$

arcsin(sin(x))=x and sin(arcsin(x))=x.

Inverse trig functions are only inverses of the trig functions on their restricted domains!

Summary

- Trigonometric functions can be used to describe rhythmic processes. To do that, one just needs to figure out the amplitude, frequency and phase shift.
- To be invertible, trig functions have to be restricted to certain domains.
- Triangles and identities are useful when simplifying inverse trigonometric relationships and calculating derivatives.

Answers

1. B
2. B
3. C