Trigonometric functions

Math 102 Section 102
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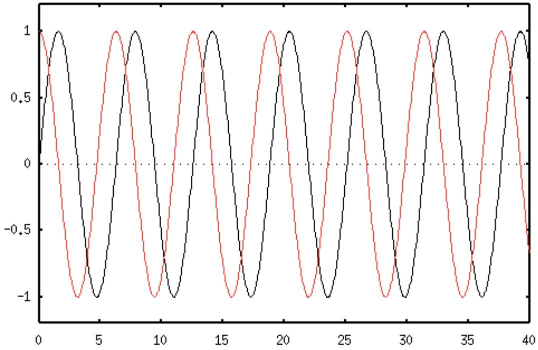
Annoucements

- Teaching evaluations
  - Before Dec. 3
  - I won't see it until your grades are finalized
  - Our Department chair, Dean of Faculty of Science will read them
  - Of course, I will read them

Today: learning goals

- Definition and properties of trigonometric functions

Romeo and Juliet



Time t →

Mathematics can be used to study social relationships, including love! If you are curious, check out this book:

Strogatz, S. H. (2014). Nonlinear dynamics and chaos: with applications to physics, biology, chemistry, and engineering. Westview press.

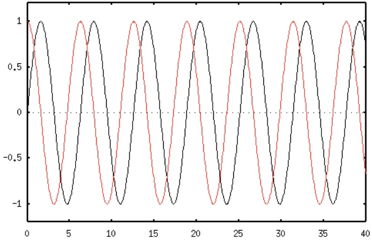
Two interesting functions

Q1. Do you recognize these functions?

Yes! These are

A. Polynomials
B. Exponentials
C. Power functions
D. Sine and cosine
E. Not sure

Romeo and Juliet



Time t →

These curves are $x(t) = \cos(t)$, $y(t) = \sin(t)$.

Introducing the trigonometric functions

- $\cos t$
- $\sin t$

What is special about these functions?

- They are periodic
- They describe oscillating systems
- They have “nice” derivatives

Derivative of cosine and sine

Cosine:

$$\frac{d}{dt}\cos t = -\sin t$$

Sine:

$$\frac{d}{dt}\sin t = \cos t$$

More to come later

Wait, what?

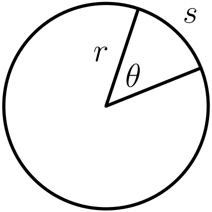
- I thought trig functions had to do with angles and triangles.
- They do!

Angles in radians

- Define a new measure for angles:

$$1 \text{ revolution around a cirle} = 2\pi \text{ radians}$$

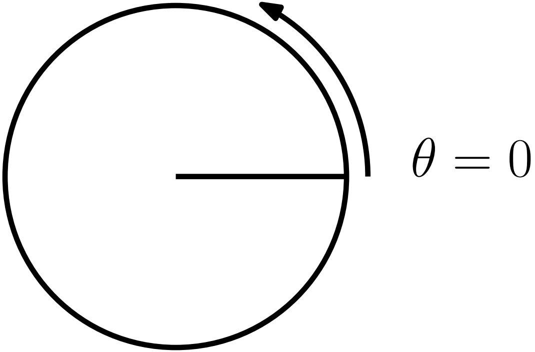
- Angles are associated with the length of an arc subtended by that angle:



$$s = r\theta$$

Convention

- Angles increase counterclockwise



Convert from degrees to radians

Q2. In terms of radians, the angles 30, 45, 60, and 90° are

A. $\pi/6$, $\pi/4$, $\pi/3$, $\pi/2$

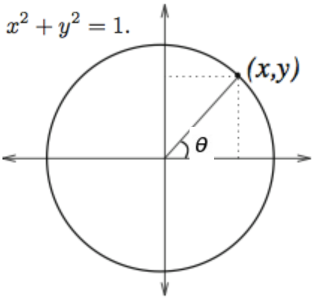
B. $\pi/3$, $\pi/2$, $\pi/6$, π

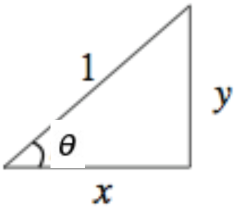
C. $\pi/30$, $\pi/45$, $\pi/60$, $\pi/90$

Special angles

degrees	radians	$\sin(t)$	$\cos(t)$	$\tan(t)$
0	0	0	1	0
30	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90	$\frac{\pi}{2}$	1	0	∞

Connection with angle (θ)





$$\sin(\theta) = \frac{y}{1} = y \qquad \cos(\theta) = \frac{x}{1} = x$$

Symmetry of trig functions

Q3. Are $\sin(t)$, $\cos(t)$ even or odd?

A. even, even

B. even, odd

C. odd, even

D. odd, odd

E. Send help!

$\sin(-t) = -\sin(t)$, $\cos(-t) = \cos(t)$

Trig identity

- Equation of circle of radius 1:

$$x^2 + y^2 = 1$$

- Point on that circle

$$(\cos(t), \sin(t))$$

- Thus,

$$\sin^2(t) + \cos^2(t) = 1$$

Two important limits

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \to 0} \frac{\cos x - 1}{x} = 0$$

Motion around a circle

- Angle increases

$$\frac{d\theta}{dt} = \omega$$

- Motion can be described by either
  1. Polar coordinates:

$$r = 1, \quad \theta(t) = \omega t$$

  2. Cartesian coordindates:

$$(x(t), y(t)) = (\cos(\omega t), \sin(\omega t))$$

Desmos demo

Periodic functions

Definition (Periodic functions)

A function is periodic with period T if for any value of t

$$f(t) = f(t + T)$$

- $\sin t$ is 2π-periodic since for any value of t

$$\sin(t + 2\pi) = \sin t$$

- same for $\cos(t)$

Frequency and period

Q4. What's the period of $\sin(\omega t)$, where $\omega \neq 0$?

A. π

B. 2π

C. $2\pi\omega$

D. $\frac{2\pi}{\omega}$

$$\sin\left(\omega(t + \frac{2\pi}{\omega})\right) = \sin(\omega t + 2\pi) = \sin(\omega t)$$

- ω is called the frequency.
- Frequency: https://www.desmos.com/calculator/n8irldojfy

Other trig functions

$$\tan t = \frac{\sin t}{\cos t}, \quad \cot t = \frac{1}{\tan t}$$
$$\sec t = \frac{1}{\cos t}, \quad \csc t = \frac{1}{\sin t}$$

Important Trigonometric Identities

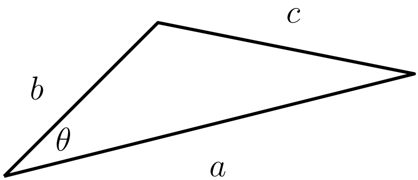
- Sum of two angles

$$\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \sin(\beta)\cos(\alpha)$$
$$\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

- Pythagorean identity

$$\sin^2(t) + \cos^2(t) = 1$$
$$\tan^2(t) + 1 = \sec^2(t)$$

Law of cosines



$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

Law of cosines

$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

Q5. In the special case, $\theta = \frac{\pi}{2}$, the law of cosines reduces to which of these?

A. $c^2 = a^2 + b^2 - 2ab$

B. $c^2 = (a - b)^2$

C. $c^2 = a^2 + b^2$

D. $\sin^2(t) + \cos^2(t) = 1$

Summary

- Trigonometric functions are related to motion around a circle
- Frequency, period, identities.

Answers

1. D
2. A
3. C
4. D
5. C

Related Exam Problems

1.8: Which of the following functions corresponds to the one plotted to the right?

(a) $f(x) = 3 + 2\sin\left(\frac{\pi}{50}(t+25)\right)$

(b) $f(x) = 5\sin\left(\frac{\pi}{100}(t+25)\right) - 1$

(c) $f(x) = 3 + 2\sin\left(\frac{\pi}{50}(t-25)\right)$

(d) $f(x) = 3 + \sin\left(\frac{\pi}{100}(t-25)\right)$

(e) $f(x) = 1 + 4\sin\left(\frac{\pi}{100}(t-25)\right)$