Disease Dynamics

Math 102 Section 102
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Due due due due due...

- Nov 19 (Today): Pre-lecture 12.1
- Nov 21 (Wednesday): Pre-lecture 12.2
- Nov 22 (Thursday): Assignment 11
- Nov 23 (Friday): OSH 6

Assignments due: 9:00 pm

Worksheet survey

Did you work on the worksheet last Friday?

A. Yes I did all of it!

B. Yes, but only part of it.

C. No I did not.

D. I did not know there was a worksheet last Friday until just now.

Results: Half of us did, and half did not. Worksheets help you get familiar with concepts and procedures that we do not have time to fully practice in class.

Epidemiology

- How does a disease spread within a population?
- Suppose that 1 infected individual is introduced to a healthy population.
  1. How many people will become infected?
  2. Will the disease persist or not?

Mathematical epidemiology

- t = time
- $S(t)$ = number of healthy people (called susceptibles)
- $I(t)$ = number of infected people
- $N(t)$ = total population

Assumptions

- Everybody mixes. Contact is random.
- Fixed probability of getting infected.
- Fixed time to get better.
- Small time scale of disease tranmission compared to population fluctuation. (N is constant.)

Keeping track of individuals

- What DE governs the rate of change of the number of infected individuals? (Document camera)

Susceptible population

If we track the size of infected population:

$$\frac{dI}{dt} = \beta SI - \mu I$$

What if tracking the size of susceptible (healty) population:
Q1. What differential equation should S satisfy?

A. $\frac{dS}{dt} = \beta SI - \mu I$
B. $\frac{dS}{dt} = -\beta SI + \mu I$
C. $\frac{dS}{dt} = \beta SI - \mu S$
D. $\frac{dS}{dt} = -\beta SI + \mu S$

Remember $S + I = N$

Disease dynamics

- Our model is a slight variant of the original model studied by Kermack and McKendrick.
- Kermack, W. O. and McKendrick, A. G. "A Contribution to the Mathematical Theory of Epidemics." Proc. Roy. Soc. Lond. A 115, 700-721, 1927.

$$\frac{dS}{dt} = -\beta SI + \mu I$$
$$\frac{dI}{dt} = \beta SI - \mu I$$

- This is called a system of differential equations.
- In our case, We can simplify the system using the algebraic relationship $N = I + S$.

Disease dynamics

$$\frac{dI}{dt} = \beta I(K - I), \qquad K = N - \frac{\mu}{\beta}.$$

Q2. Is K positive or negative?

A. Positive.

B. Negative.

C. It depends.

Qualitative analysis of the SI model

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Summary

- Law of Mass Action can be extended to "rate of change that are proportional to two things"

$$\beta SI$$

- A system of differential equations describes how quantities can change in response to each other. Sometimes they can be simplified
- Qualitative analysis (and/or Euler's Method) can be used to understand the behaviour of a system of differential equations
- The parameter K (or equivalently, the basic reproduction number $R_0 = N\beta/\mu$) quantifies whether an epidemic will occur.

Limitations to our model

- What about vaccination?
- What about immunity to the disease?
- What about virus dynamics?
- There are many other interesting questions...

Answers

1. B
2. C

Related Exam Problems

5. The model given below on the left has been suggested for the spread of HIV within the immune system of an infected person. $C(t)$ is the density of healthy immune cells, $I(t)$ is the density of HIV-infected immune cells and $V(t)$ is the density of virus in the blood of a patient. Which of the options on the right gives a correct interpretation of some part of the model?

$$\frac{dC}{dt} = P - \alpha CV - \gamma_1 C$$

$$\frac{dI}{dt} = \alpha CV - \gamma_2 I$$

$$\frac{dV}{dt} = \beta I - \gamma_3 V$$

(a) Healthy cells can become infected when they encounter infected cells.

(b) Healthy cells can become infected when they encounter virus.

(c) Virus is produced at a rate proportional to the current viral density.

(d) Infected cells die at a rate proportional to the viral density.

(e) Virus is killed/removed at a rate proportional to the density of healthy immune cells.