# 1. Law of mass action.

1 The reaction is:

$3A \to P$ That is $A + A + A \to P$

The mass balance:

rate of change of concentration of A

= rate of addition − rate of consumption

$\downarrow$ known, $I$

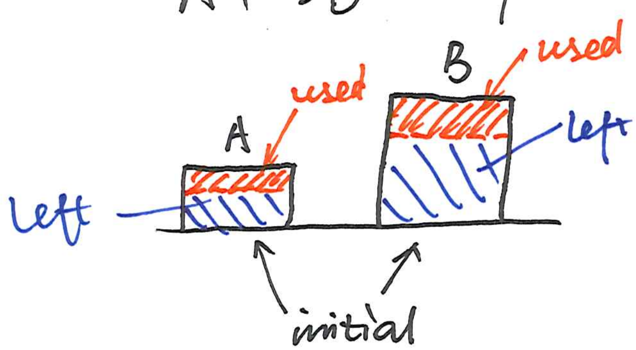
$\downarrow$ use Law of Mass Action.

$= k \cdot a \cdot a \cdot a = ka^3$

$$\Rightarrow \frac{da}{dt} = I - ka^3.$$

2 The reaction is:

$A + 2B \to P$




Since A starts out as half of B, and $A + 2B \to P$, we have

$a(t) = \frac{1}{2} b(t)$ ← concentration of B

at any time $t$.

$\Rightarrow b(t) = 2a(t)$

Mass balance:

$$\frac{da}{dt} = 0 - k a \cdot b \cdot b = -kab^2 = -4ka^3$$

2. Logistic growth.

$$\frac{dN}{dt} = rN(1 - \frac{N}{K})$$

$$= rN - r\frac{N^2}{K}$$

When $N$ is big, $\underline{\underline{-r\frac{N^2}{K}}}$ dominates, $\frac{dN}{dt} < 0$.

↳ ~~controls~~ suppresses over-growth.

3. Scaling

Let $\underbrace{\tau = \frac{t}{\frac{1}{r}}}_{\text{measure time in the unit of } \frac{1}{r}}$, $\underbrace{y = \frac{N}{K}}_{\text{measure population size in the unit of } K.}$

$$\Rightarrow t = \frac{\tau}{r}, \quad N = Ky$$

$$\frac{dN}{dt} = \lim_{\Delta t \to 0} \frac{\Delta N}{\Delta t} = \lim_{\Delta \tau \to 0} \frac{K \Delta y}{\frac{1}{r} \Delta \tau} = Kr \lim_{\Delta \tau \to 0} \frac{\Delta y}{\Delta \tau}$$

$$= Kr \frac{dy}{d\tau}$$

$$\Rightarrow \cancel{K}\cancel{r}\frac{dy}{d\tau} = r\cancel{K}y(1-y)$$

$$\frac{dy}{d\tau} = y(1-y)$$

A new DE without any parameter!

If we can ~~solve~~ solve this DE, we can back out any N given any values of K, r.

One equation rules all.

dimensionless dimensional