Power functions and polynomials
Math 102 Section 102

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Sep. 7, 2018

Announcements

- WeBWorK assignments for next week!
- New Office hours: T 2:30-4:00 W 3:00-4:00 @LSK 300
- Office hours Today: 3:00-4:00 @LSK 300
- MLC: open from Sep. 14

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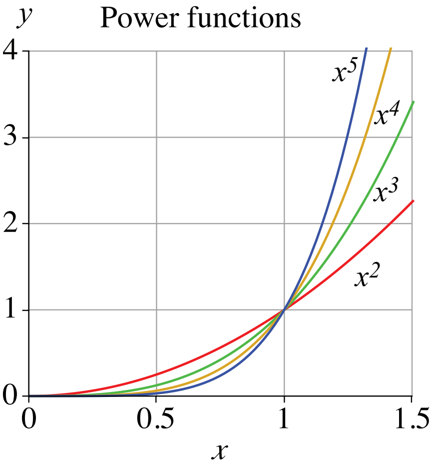
Today

- Power functions cont'd
- Even and odd functions
- Polynomials and sketching their graph

Last time: asymptotic thinking

- Small powers dominate close to $x = 0$; large powers dominate for large x.



Domination

Fact (Domination of power functions)

When a power function ax^n dominates another bx^m in a range of x, that means

$$ax^n + bx^m \approx ax^n$$

in that range of x (asymptotically).

Example

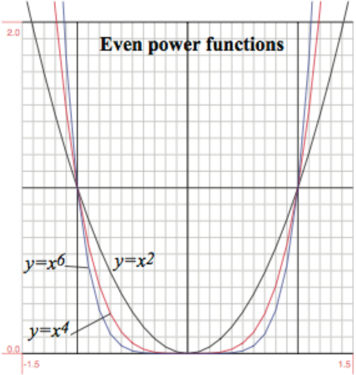
How does $2x^4 + 0.5x^2$ behave close to $x = 0$ and at large x values? Demo

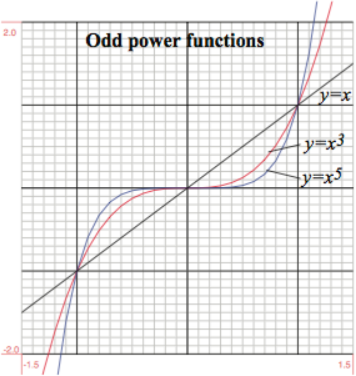
Polynomials

Definition (Polynomials)

A polynomial is a sum of power functions.

Even and odd functions





Even and odd functions

Definition (Even and odd functions)

- $f(x)$ is an even function if it is symmetric about the y-axis or
$$f(-x) = f(x), \quad \forall x;$$
($\forall =$ for all)
- $f(x)$ is an odd function if it is symmetric about the origin or
$$f(-x) = -f(x), \quad \forall x.$$

Even and odd functions

Example

The function $y = f(x) = C$ for C constant is an even function

- Geometric argument: a constant function is symmetric about the y-axis.
- Algebraic argument: $f(-x) = C = f(x)$

Even and odd functions

Q1. The product of two odd functions is

A. an odd function

B. an even function

C. both even and odd

D. neither even nor odd

E. not enough information to tell

- Let $h(x) = f(x)g(x)$ with $f,\ g$ being odd functions,

$$\begin{aligned} h(-x) &= f(-x)g(-x) \\ &= (-1)^2 f(x)g(x) \\ &= f(x)g(x) = h(x). \end{aligned}$$

Even and odd functions

Q2. The function $f(x) = \frac{x^2}{1+x^2}$ (the quotient of two polynomials is called a rational function) is

A. an odd function

B. an even function

C. both even and odd

D. neither even nor odd

E. not enough information to tell

Even and odd functions

Q3. The function $g(x) = \frac{x^3}{1+x^3}$ is

A. an odd function

B. an even function

C. both even and odd

D. neither even nor odd

E. not enough information to tell

Even and odd functions

- If $f(x)$ and $g(x)$ are both even functions then $f+g$, $f-g$, fg, and f/g are all even functions.
- If $f(x)$ and $g(x)$ are both odd functions then $f+g$ and $f-g$ are odd functions, but fg and f/g are even functions.

For you to think about: Why is this true?

Sketching a polynomial

- Goal: to be able to easily sketch the graph of a simple polynomial function

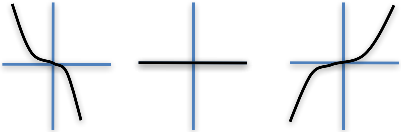
$$f(x) = ax^n + bx^m.$$

- Key idea:
  - Lower powers dominate near $x = 0$.
  - Higher powers dominate for x far from 0.
  - Use symmetry (even and odd power functions).

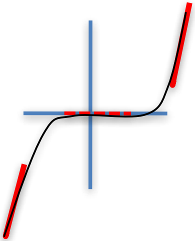
Sketching a polynomial

Example (Sketch $y = x^5 + ax^3$.)

- Near $x = 0$, $y \approx ax^3$. $a < 0$, $a = 0$, $a > 0$:



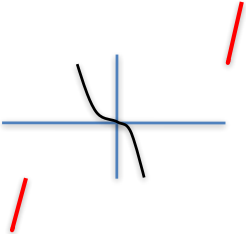
- Far from $x = 0$, $y \approx x^5$:



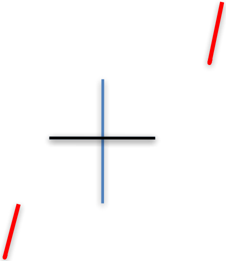
Sketching a polynomial

Example (Sketch $y = x^5 + ax^3$.)

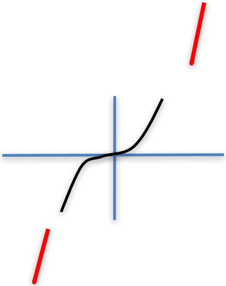
$a < 0$:



$a = 0$:



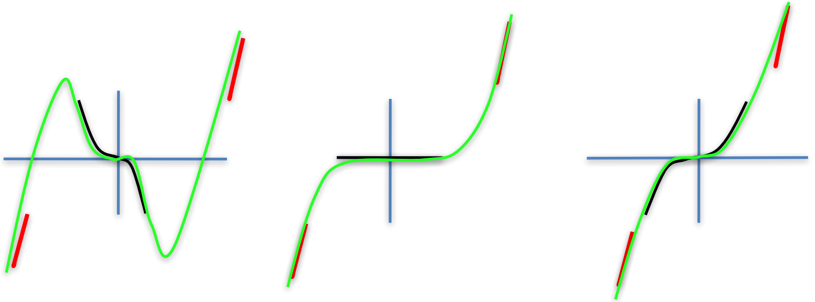
$a > 0$:



Sketching a polynomial

Example (Sketch $y = x^5 + ax^3$.)

$a < 0$: $a = 0$: $a > 0$:



Zeros are the x locations where the function value becomes 0.

Finding the zeros

Q4. Suppose $a = -4$, with $y = x^5 + ax^3$. The zeros of $y = x^5 - 4x^3$ are:

A. $x = 2$

B. $x = \pm 4$

C. $x = \pm 2$

D. $x = 0, \pm 2$

E. $x = 0, \pm\sqrt{2}$

Power functions and curve sketching

Q5. Which of the functions below has this graph?

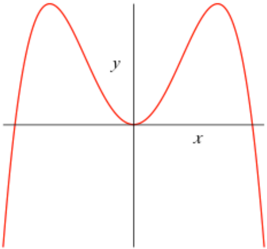
A. $x^3 - x^5$

B. $x^5 - x^3$

C. $x^4 + x^2$

D. $x^4 - x^2$

E. $x^2 - x^4$



- Step 1: odd or even, or neither?
- Step 2: asymptotic behavior

Today...

- Power functions and domination
- Polynomials: $f(x) = ax^n + bx^m$
- Rational functions: $g(x) = \frac{ax^n + bx^m}{cx^\ell + dx^k}$
- Even functions vs. odd functions
- Sketching the graph of simple polynomials:
  - Large powers away from $x = 0$
  - Small powers near $x = 0$
  - Connecting different parts smoothly
  - Identify the zeros
- Check the last slides for sample exam problems

Answers

1. B
2. B
3. D
4. D
5. E

Sample Exam Questions

1. When $x = 1000$, the function $g(x) = \frac{6x^4+12x^2+64x-87}{2x^3-6x^2+x}$ is closet to
   - A. 0.003
   - B. 3000
   - C. 1000000
   - D. 6
   - E. 3
2. Sketch the graph of $f(x) = 8x^2 - x^5$.